Final Exam	Time: 120 min
Course: *58:160, Fall 2006*	
Name:-------------------------	**Fluid I.D. :**-----------

Solutions:

1)

$\tau_w = \rho u*^2 \Rightarrow 122 = 998u*^2 \Rightarrow u* = 0.35 m/s$

a) $V_C = u*\left(\frac{1}{\kappa}\ln\left(\frac{Ru*}{\nu}\right) + B\right) = 0.35\left(\frac{1}{0.41}\ln\left(\frac{998*0.045*0.35}{0.001}\right) + 5\right) = 10 m/s$

b) $V = u*\left(2.44\ln\left(\frac{Ru*}{\nu}\right) + 1.34\right) = 8.72 m/s$

$Q = AV = 0.055$

c) $\Delta P = 2\tau_w \Delta L / R = 542000 Pa$

2)

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence ε/d = 0.26/50 ≈ 0.0052. The minor loss coefficients are Entrance: K ≈ 0.5; 5-cm(≈2″) open globe valve: K ≈ 6.9

The flow rate is known, hence we can compute V, Re, and f:

$$V = \frac{Q}{A} = \frac{0.004}{(\pi/4)(0.05)^2} = 2.04\ \frac{m}{s},\quad Re = \frac{998(2.04)(0.05)}{0.001} \approx 102000,\quad f \approx 0.0316$$

The turbine head equals the elevation difference minus losses and exit velocity head:

$$h_t = \Delta z - h_f - \sum h_m - \frac{V^2}{2g} = 40 - \frac{(2.04)^2}{2(9.81)}\left[(0.0316)\left(\frac{125}{0.05}\right) + 0.5 + 6.9 + 1\right] \approx 21.5\ m$$

$$\text{Power} = \rho g Q h_t = (998)(9.81)(0.004)(21.5) \approx \mathbf{840\ W} \quad \textit{Ans.}$$

3) $\sum F_z = 0 \Rightarrow T + D + F_B = W$

Where:

T=tension force

$F_{B=}$ Bouyancy Force

D=friction Drag

W= weight

$F_B = \rho_w g * V = 9810*0.003*1*3 = 88.3N$

$D = C_D(0.5\rho A V^2)$

$\mathrm{Re} = \rho VL/\mu = 4.6e6$ Turbulent flow

$C_D = 0.031/\mathrm{Re}^{1/7}$ =3.46e-3

$D = 0.003*2*3*1000*36/2 = 324N$

$T = 500 - 324 - 88.3 = 87.4N$

4)

$$\Sigma M_o = F_a(0.1\sin 45°) - W_a(0.1\cos 45°) - F_b(0.1\sin 45°) + W_b(0.1\cos 45°) = 0$$

For 45°, there are nice cancellations to obtain $\therefore$ $F_a - F_b = W_a - W_b$, or:

$$C_{Da}\frac{\rho}{2}U^2\frac{\pi}{4}D_a^2 - C_{Db}\frac{\rho}{2}U^2\frac{\pi}{4}D_b^2 \stackrel{?}{=} (SG)\rho_{water}g\frac{\pi}{6}D_a^3 - (SG)\rho_{water}g\frac{\pi}{6}D_b^3$$

Assuming that $C_{Da} = C_{Db} \approx 0.47$ (Re < 250000), we may easily solve for air velocity:

$$U^2(0.47)\left(\frac{1.225}{2}\right)\frac{\pi}{4}[(0.02)^2 - (0.01)^2] = (7.86)(9790)\frac{\pi}{6}[(0.02)^3 - (0.01)^3]$$

Solve for $U = \sqrt{4158} \approx$ **64 m/s** *Ans.*

We may check that $Re_{max} = 1.225(64)(0.02)/1.78E{-5} \approx$ *89000*, OK, $C_D \approx 0.47$.

5)

$$\psi = U_\infty \sin\theta(r - a^2/r) = U_\infty y - U_\infty y\frac{a^2}{x^2+y^2} = U_\infty y(1 - \frac{a^2}{x^2+y^2})$$

At $x = \infty$: y=h
At $x = 0$: y=h+a/3
Those points are on the same streamline:

a) $\psi = U_\infty h = U_\infty (h + a/3)(1 - \frac{a^2}{(h+a/3)^2}) \Rightarrow h = \frac{8}{3}a$

b) $U_{max} = U_\infty \sin 90(1 + \frac{a^2}{(h+a/3)^2}) = U_\infty(1 + \frac{a^2}{(3a)^2}) = \frac{10}{9}U_\infty$