1) **Total (15)**

Table A-5 : at 35°C the vapor pressure of water is approximately 5800 Pa. **(1)**
Table A-1 : at 35°C the density of water is 994 kg/m³ or approximately 998 kg/m³

Bernoulli from the surface to point 3 gives:

(4) **(1)**

$$\left.\begin{array}{l} \frac{P_s}{\gamma}+\frac{V_s^2}{2g}+z_s=\frac{P_3}{\gamma}+\frac{V_3^2}{2g}+z_3 \\ P_s=P_3=P_a \\ V_s=0 \\ z_s=h \\ z_3=0 \end{array}\right\} \Rightarrow V_3^2=2gh=2\times 32.2\times 6=386.4 \Rightarrow V_3=19.66\,ft/s=5.99\,m/s$$

By continuity:

(2) **(1)**

$$V_1A_1=V_3A_3 \Rightarrow V_1=\frac{A_3}{A_1}V_3=(\frac{D}{1/12})^2\times 19.66=2831.04D^2\ English\ Sys$$

$$V_1A_1=V_3A_3 \Rightarrow V_1=\frac{A_3}{A_1}V_3=(\frac{D}{0.3048/12})^2\times 5.99=9284.52D^2\ Metric\ Sys$$

Bernoulli from point 1 to point 3 gives:

(3) **(1)**

$$\left.\begin{array}{l} \frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_3}{\gamma}+\frac{V_3^2}{2g}+z_3 \\ P_3=P_a=100KPa \\ P_1=P_v=5800Pa \\ V_1=9284.52D^2 \\ V_3=5.99m/s \\ z_1=z_3=0 \\ \gamma=994\times 9.81=9751.14 \end{array}\right\} \Rightarrow D^4=2.61e-6 \Rightarrow D=0.0402m=0.132\,ft$$

By looking carefully the above Bernoulli equation:

(1)

$$\left.\begin{aligned}&\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_3}{\gamma}+\frac{V_3^2}{2g}+z_3\\&P_3=P_a\\&V_1=9284.52D^2\\&V_3=5.99m/s\\&z_1=z_3=0\end{aligned}\right\}\Rightarrow\frac{P_1}{\gamma}=\frac{P_a}{\gamma}+\frac{5.99^2}{2g}-\frac{(9284.52D^2)^2}{2g}$$

(1)

To avoid cavitation, we would decrease D which will increase P_1 to keep $P_1 > P_v$.

2) **Total (15)**

a) First examine the momentum of the jet striking the plate:

(4) **(1)**

$$\sum F_x=\dot{m}_{out}v_{out}-\dot{m}_{in}v_{in}$$

$$-F=0-\rho A_2V_2^2\Rightarrow 70=998\times\frac{\pi}{4}\times0.03^2\times V_2^2\Rightarrow V_2=9.96m/s$$

By continuity:

(2) **(1)**

$$V_1A_1=V_2A_2\Rightarrow V_1=\frac{A_2}{A_1}V_2=(\frac{3}{10})^2\times9.96=0.9m/s$$

b) Applying Bernoulli:

(3) **(1)**

$$\left.\begin{aligned}&\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2\\&V_1=0.9\\&V_2=9.96\\&z_1=z_3=0\end{aligned}\right\}\Rightarrow P_2-P_1=\frac{1}{2}\rho\left(V_2^2-V_1^2\right)=\frac{1}{2}\times998\times\left(9.96^2-0.9^2\right)=49097.4Pa$$

And from manometer principles:

(2) **(1)**

$$P_2+\gamma_{water}h=P_1+\gamma_{mercury}h$$

$$h=\frac{\Delta P}{(\rho_{mercury}-\rho_{water})g}=\frac{49097.4}{(13600-998)\times9.81}=0.4m$$

3) **Total(10)**

Applying x-momentum equation:

(1)

$$\sum F_x = \dot{m}_{out} v_{out} - \dot{m}_{in} v_{in} = \dot{m}(V_2 - V_1)$$

We have just pressure force per unit width:

(2)

$$LHS = P_1 A_1 - P_2 A_2 = \frac{1}{2}\rho g h_1 (h_1) - \frac{1}{2}\rho g h_2 (h_2)$$

$$RHS = \dot{m}(V_2 - V_1) = \rho h_1 V_1 (V_2 - V_1)$$

(1)

$$\Rightarrow \frac{1}{2}\rho g h_1 (h_1) - \frac{1}{2}\rho g h_2 (h_2) = \rho h_1 V_1 (V_2 - V_1)$$

$$\frac{1}{2} g \left(h_1^2 - h_2^2\right) = h_1 V_1 (V_2 - V_1) \quad (1)$$

Applying continuity:

(2)

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho A_1 V_1 = \rho A_2 V_2 \Rightarrow h_1 V_1 = h_2 V_2 \Rightarrow \frac{V_2}{V_1} = \frac{h_1}{h_2} \quad (2)$$

By mixing (1) and (2):

(1)

$$\frac{1}{2} g \left(h_1^2 - h_2^2\right) = h_1 V_1 \left(\frac{h_1}{h_2} V_1 - V_1\right)$$

$$= \frac{h_1}{h_2} V_1^2 \left(h_1 - h_2\right)$$

$$\Rightarrow \frac{1}{2} g \left(h_1 + h_2\right)\left(h_1 - h_2\right) = \frac{h_1}{h_2} V_1^2 \left(h_1 - h_2\right)$$

$$\left(h_1 + h_2\right) = \frac{2h_1}{g h_2} V_1^2$$

$$h_2^2 + h_1 h_2 - \frac{2h_1}{g} V_1^2 = 0 \qquad (3)$$

$$h_2 = \frac{-h_1 - \sqrt{h_1^2 + \frac{8h_1}{g} V_1^2}}{2} < 0 \; Unacceptable$$

(1)

$$h_2 = \frac{-h_1 + \sqrt{h_1^2 + \frac{8h_1}{g} V_1^2}}{2}$$ **Ans(a)**

$$V_2 = V_1 \frac{h_1}{h_2} = \frac{V_1}{-1/2 + 1/2\sqrt{1 + \frac{8}{gh_1} V_1^2}}$$ **Ans(a)**

b) Applying energy equation from point 1 to point 2 gives: (both points are on free surface)

(1)

$$\left.\begin{array}{l} \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + H_L \\ P_1 = P_2 = 0 \\ z_1 = h_1 \\ z_2 = h_2 \end{array}\right\} \Rightarrow H_L = \frac{V_1^2}{2g}\left(1 - \frac{V_2^2}{V_1^2}\right) + h_1 - h_2$$

By using equations (2) and (3):

$$H_L = \frac{V_1^2}{2g}\left(1 - \frac{V_2^2}{V_1^2}\right) + h_1 - h_2 = \frac{h_2^2 + h_1 h_2}{4h_1}\left(1 - \left(\frac{h_1}{h_2}\right)^2\right) + h_1 - h_2$$

$$= \frac{(h_2 + h_1)h_2}{4h_1 h_2^2}\left(h_2^2 - h_1^2\right) + h_1 - h_2$$

$$= (h_2 - h_1)\left\{\frac{(h_2 + h_1)^2}{4h_1 h_2} - 1\right\}$$

$$= \frac{(h_2 - h_1)}{4h_1 h_2}\left(h_2^2 + h_1^2 - 2h_1 h_2\right)$$

$$= \frac{(h_2 - h_1)}{4h_1 h_2}(h_2 - h_1)^2$$

(1)

$$\Rightarrow H_L = \frac{(h_2 - h_1)^3}{4h_1 h_2}$$ **Ans b**