

1) **Total (15)**

Table A-5 : at 35°C the vapor pressure of water is approximately 5800 Pa. **(1)**

Table A-1 : at 35°C the density of water is 994 kg/m³ or approximately 998 kg/m³

Bernoulli from the surface to point 3 gives:

$$\begin{array}{l}
 \xleftarrow{(4)} \qquad \qquad \qquad \xrightarrow{(1)} \\
 \left. \begin{array}{l}
 \frac{P_s}{\gamma} + \frac{V_s^2}{2g} + z_s = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \\
 P_s = P_3 = P_a \\
 V_s = 0 \\
 z_s = h \\
 z_3 = 0
 \end{array} \right\} \Rightarrow V_3^2 = 2gh = 2 \times 32.2 \times 6 = 386.4 \Rightarrow V_3 = 19.66 \text{ ft/s} = 5.99 \text{ m/s}
 \end{array}$$

By continuity:

$$\begin{array}{l}
 \xleftarrow{(2)} \qquad \qquad \qquad \xrightarrow{(1)} \\
 V_1 A_1 = V_3 A_3 \Rightarrow V_1 = \frac{A_3}{A_1} V_3 = \left(\frac{D}{1/12}\right)^2 \times 19.66 = 2831.04 D^2 \text{ English Sys} \\
 V_1 A_1 = V_3 A_3 \Rightarrow V_1 = \frac{A_3}{A_1} V_3 = \left(\frac{D}{0.3048/12}\right)^2 \times 5.99 = 9284.52 D^2 \text{ Metric Sys}
 \end{array}$$

Bernoulli from point 1 to point 3 gives:

$$\begin{array}{l}
 \xleftarrow{(3)} \qquad \qquad \qquad \xrightarrow{(1)} \\
 \left. \begin{array}{l}
 \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \\
 P_3 = P_a = 100 \text{ KPa} \\
 P_1 = P_v = 5800 \text{ Pa} \\
 V_1 = 9284.52 D^2 \\
 V_3 = 5.99 \text{ m/s} \\
 z_1 = z_3 = 0 \\
 \gamma = 994 \times 9.81 = 9751.14
 \end{array} \right\} \Rightarrow D^4 = 2.61e-6 \Rightarrow D = 0.0402 \text{ m} = 0.132 \text{ ft}
 \end{array}$$

By looking carefully the above Bernoulli equation:

$$\begin{array}{l}
 \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \\
 P_3 = P_a \\
 V_1 = 9284.52D^2 \\
 V_3 = 5.99m/s \\
 z_1 = z_3 = 0
 \end{array}
 \left. \vphantom{\begin{array}{l} \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \\ P_3 = P_a \\ V_1 = 9284.52D^2 \\ V_3 = 5.99m/s \\ z_1 = z_3 = 0 \end{array}} \right\} \Rightarrow \frac{P_1}{\gamma} = \frac{P_a}{\gamma} + \frac{5.99^2}{2g} - \frac{(9284.52D^2)^2}{2g}$$

$$\overleftarrow{\hspace{15em}} \quad \text{(1)} \quad \overrightarrow{\hspace{15em}}$$

To avoid cavitation, we would decrease D which will increase P_1 to keep $P_1 > P_v$.

2) Total (15)

a) First examine the momentum of the jet striking the plate:

$$\begin{array}{l}
 \overleftarrow{\hspace{5em}} \quad \text{(4)} \quad \overrightarrow{\hspace{10em}} \quad \text{(1)} \\
 \sum F_x = \dot{m}_{out} v_{out} - \dot{m}_{in} v_{in} \\
 -F = 0 - \rho A_2 V_2^2 \Rightarrow 70 = 998 \times \frac{\pi}{4} \times 0.03^2 \times V_2^2 \Rightarrow V_2 = 9.96m/s
 \end{array}$$

By continuity:

$$\begin{array}{l}
 \overleftarrow{\hspace{5em}} \quad \text{(2)} \quad \overrightarrow{\hspace{10em}} \quad \text{(1)} \\
 V_1 A_1 = V_2 A_2 \Rightarrow V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{3}{10}\right)^2 \times 9.96 = 0.9m/s
 \end{array}$$

b) Applying Bernoulli:

$$\begin{array}{l}
 \overleftarrow{\hspace{15em}} \quad \text{(3)} \quad \overrightarrow{\hspace{10em}} \quad \text{(1)} \\
 \left. \begin{array}{l} \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \\ V_1 = 0.9 \\ V_2 = 9.96 \\ z_1 = z_2 = 0 \end{array} \right\} \Rightarrow P_2 - P_1 = \frac{1}{2} \rho (V_2^2 - V_1^2) = \frac{1}{2} \times 998 \times (9.96^2 - 0.9^2) = 49097.4Pa
 \end{array}$$

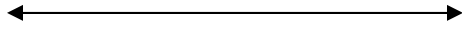
And from manometer principles:

$$\begin{array}{l}
 \overleftarrow{\hspace{5em}} \quad \text{(2)} \quad \overrightarrow{\hspace{10em}} \quad \text{(1)} \\
 P_2 + \gamma_{water} h = P_1 + \gamma_{mercury} h \\
 h = \frac{\Delta P}{(\rho_{mercury} - \rho_{water})g} = \frac{49097.4}{(13600 - 998) \times 9.81} = 0.4m
 \end{array}$$

3) **Total(10)**

Applying x-momentum equation:

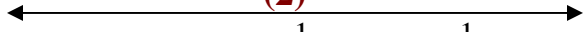
(1)



$$\sum F_x = \dot{m}_{out} v_{out} - \dot{m}_{in} v_{in} = \dot{m}(V_2 - V_1)$$

We have just pressure force per unit width:

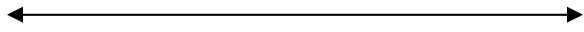
(2)



$$LHS = P_1 A_1 - P_2 A_2 = \frac{1}{2} \rho g h_1 (h_1) - \frac{1}{2} \rho g h_2 (h_2)$$

$$RHS = \dot{m}(V_2 - V_1) = \rho h_1 V_1 (V_2 - V_1)$$

(1)



$$\Rightarrow \frac{1}{2} \rho g h_1 (h_1) - \frac{1}{2} \rho g h_2 (h_2) = \rho h_1 V_1 (V_2 - V_1)$$

$$\frac{1}{2} g (h_1^2 - h_2^2) = h_1 V_1 (V_2 - V_1) \quad (1)$$

Applying continuity:

(2)



$$\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho A_1 V_1 = \rho A_2 V_2 \Rightarrow h_1 V_1 = h_2 V_2 \Rightarrow \frac{V_2}{V_1} = \frac{h_1}{h_2} \quad (2)$$

By mixing (1) and (2):

(1)



$$\frac{1}{2} g (h_1^2 - h_2^2) = h_1 V_1 \left(\frac{h_1}{h_2} V_1 - V_1 \right)$$

$$= \frac{h_1}{h_2} V_1^2 (h_1 - h_2)$$

$$\Rightarrow \frac{1}{2} g (h_1 + h_2) (h_1 - h_2) = \frac{h_1}{h_2} V_1^2 (h_1 - h_2)$$

$$(h_1 + h_2) = \frac{2h_1}{gh_2} V_1^2$$

$$h_2^2 + h_1 h_2 - \frac{2h_1}{g} V_1^2 = 0 \quad (3)$$

$$h_2 = \frac{-h_1 - \sqrt{h_1^2 + \frac{8h_1}{g} V_1^2}}{2} < 0 \text{ Unacceptable}$$

(1)

$$\begin{array}{c}
 \leftarrow \hspace{10em} \rightarrow \\
 h_2 = \frac{-h_1 + \sqrt{h_1^2 + \frac{8h_1}{g}V_1^2}}{2} \quad \text{Ans(a)}
 \end{array}$$

$$V_2 = V_1 \frac{h_1}{h_2} = \frac{V_1}{-1/2 + 1/2 \sqrt{1 + \frac{8}{gh_1}V_1^2}} \quad \text{Ans(a)}$$

b) Applying energy equation from point 1 to point 2 gives: (both points are on free surface)

(1)

$$\begin{array}{c}
 \leftarrow \hspace{10em} \rightarrow \\
 \left. \begin{array}{l}
 \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + H_L \\
 P_1 = P_2 = 0 \\
 z_1 = h_1 \\
 z_2 = h_2
 \end{array} \right\} \Rightarrow H_L = \frac{V_1^2}{2g} \left(1 - \frac{V_2^2}{V_1^2} \right) + h_1 - h_2
 \end{array}$$

By using equations (2) and (3):

$$\begin{aligned}
 H_L &= \frac{V_1^2}{2g} \left(1 - \frac{V_2^2}{V_1^2} \right) + h_1 - h_2 = \frac{h_2^2 + h_1 h_2}{4h_1} \left(1 - \left(\frac{h_1}{h_2} \right)^2 \right) + h_1 - h_2 \\
 &= \frac{(h_2 + h_1)h_2}{4h_1 h_2^2} (h_2^2 - h_1^2) + h_1 - h_2 \\
 &= (h_2 - h_1) \left\{ \frac{(h_2 + h_1)^2}{4h_1 h_2} - 1 \right\} \\
 &= \frac{(h_2 - h_1)}{4h_1 h_2} (h_2^2 + h_1^2 - 2h_1 h_2) \\
 &= \frac{(h_2 - h_1)}{4h_1 h_2} (h_2 - h_1)^2
 \end{aligned}$$

(1)

$$\begin{array}{c}
 \leftarrow \hspace{10em} \rightarrow \\
 \Rightarrow H_L = \frac{(h_2 - h_1)^3}{4h_1 h_2} \quad \text{Ans b}
 \end{array}$$