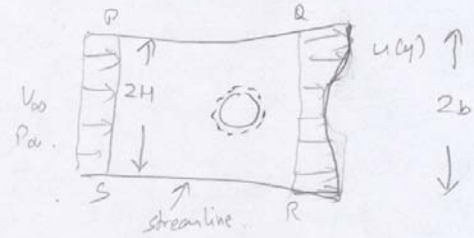
1)



← 8 pts

find relation between H & b using mass balance

$$2 H V_\infty \rho = 2 \rho \int_0^b u(y)\, dy$$ ← 6 pts

$$H = \frac{1}{V_\infty} \int_0^b u(y)\, dy$$ ← 2 pts

Momentum eqn

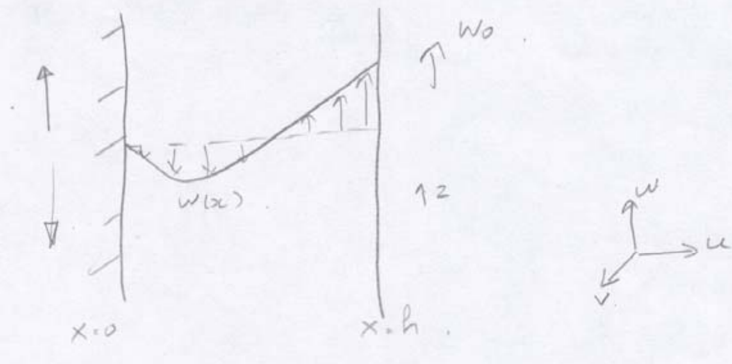
$$-f_x = 2\rho \int_0^b u(y)^2\, dy - 2\rho H V_\infty^2$$ ← 7 pts

$$= 2\rho \int_0^b u(y)^2\, dy - 2\rho V_\infty \int_0^b u(y)\, dy$$

$$= 2\rho \int_0^b u\,(u - V_\infty)\, dy$$ ← 2 pts

$$\therefore F_x = 2\rho \int_0^b u(U_\infty - u)\, dy$$

2)



Assumptions: $u = v = 0$ $\quad w = w(x)$.

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0 .$$

(Problem set-up. + 5 pts)

BCs $\quad u, v, w \ (\text{at } x=0) = 0$

$\quad w = w_0 \text{ at } (x=h)$

z- momentum eqn.

$$\underbrace{\frac{\partial w}{\partial t}}_{0} + \underbrace{u\frac{\partial w}{\partial x}}_{0} + \underbrace{v\frac{\partial w}{\partial y}}_{0} + \underbrace{w\frac{\partial w}{\partial z}}_{0} = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial z}}_{0} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \underbrace{\frac{\partial^2 w}{\partial y^2}}_{0} + \cancel{\frac{\partial^2 w}{\partial z^2}}\right)$$

$$\boxed{\frac{\partial^2 w}{\partial x^2} = -\frac{g_z}{\nu}}$$

(5 pts)

$$\frac{\partial w}{\partial x} = \frac{-g_z}{\nu} x + C_1$$

a) $$w = \frac{-g_z}{\nu}\frac{x^2}{2} + C_1 x + C_2$$

↑ 5 pts.

At $x=0$ $w=0$.

$$\boxed{C_2 = 0}$$

At $x = h$ $w = w_0$.

Applying ← B.Cs. 2pts

$$w_0 = -\frac{g_z h^2}{2\nu} + C_1 h.$$

$$\boxed{C_1 = \frac{w_0}{h} + \frac{g_z h}{2\nu}}$$

b) w at $x = h/2$

$$w_{(h/2)} = \frac{-g_z}{\nu}\frac{h^2}{8} + \frac{w_0}{h}\frac{h}{2} + \frac{g_z}{\nu}\frac{h}{2}\frac{h}{2}$$

$$= \frac{1}{2}\left(w_0 - \frac{8gh^2}{4\nu}\right)$$

3 ($\cancel{8}$)

$$\frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$$

c) Mass flow : $\int_0^h \rho h\, w(x)\, dx = 0$. $\leftarrow \boxed{5}$

$$\rho h \int_0^h \left(\frac{-g_z}{\nu} \frac{x^2}{2} + \frac{w_0 x}{h} + \frac{g_z h x}{2\nu} \right) dx$$

$$\rho h \left| \frac{-g_z}{2\nu} \frac{x^3}{3} + \frac{w_0}{h} \frac{x^2}{2} + \frac{g h}{2\nu} \frac{x^2}{2} \right|_0^h$$

$$\rho h \left[\frac{g_z h^3}{6\nu} + \frac{w_0 h}{2} + \frac{g h^3}{2 \cdot 2\nu} \right] = 0$$

$\cancel{5}$

$$w_0 = \frac{\rho g h^2}{6\mu}$$

3)

a) 3 independent ~~dimensionless groups~~.

$$f = \phi\,(D, L, V, \nu) \quad \longleftarrow \quad 5$$

$n = 5$

$j = 2$

$K = 3$

a)

$$\Pi_1 = \frac{fD}{V} \quad \longleftarrow \quad 3$$

$$\Pi_2 = \frac{VD}{\nu} \quad \longleftarrow \quad 3$$

$$\Pi_3 = \frac{L}{D} \quad \longleftarrow \quad 1$$

b) i) Equate Re

$$\frac{V_m D_m}{\nu_m} = \frac{V_p D_p}{\nu_p} \quad \longleftarrow \quad 5$$

$$V_m = 10\,V_p \quad \longleftarrow \quad 1.5$$

ii) Equate St. no

$$\frac{f_m D_m}{V_m} = \frac{f_p D_p}{V_p} \quad \longleftarrow \quad 5$$

$$f_m = 100 f_p \quad \longleftarrow \quad 1.5$$

4) $Re = \frac{VD}{\nu} = 10^6$ (So turbulent) 5

find f from moody chart $f = 0.0116$. -3

$$\left[\begin{array}{ll} \text{find } \tau_w \text{ from } f & -2 \\ & + \\ \text{find } u^* \text{ from } \tau_w & \underline{-2} \\ & 4 \end{array}\right]$$

total 7

[or] or

find u^+ from $f = 4$.
$(u^* = V(\frac{f}{8})^{1/2})$

[or]
find u^* from [eqn. 6.34] -7
without finding f

a) laminar sublayer $y^+ = 5$ (1)
$y^+ = \frac{yu^*}{\nu}$ $y = 2.6 \times 5 \times 10^{-6}$ (1) 2

b) Turbulent core $y^+ = 30$
$y^+ = \frac{yu^*}{\nu}$ $y = 0.078$ mm 2

c) use log law for profile -5
use linear $y^+ = u^+$ for laminar sublayer -1

d) Substitution 1 pts x 3 → 3 pts.