1) The viscous sublayer is normally less than 1 percent of the pipe diameter and therefore very difficult to probe with a finite-sized instrument. In an effort to generate a thick sublayer for probing, Pennsylvania State University in 1964 built a pipe with a flow of glycerin. Assume a smooth 12-in-diameter pipe with $V = 60$ ft/s and glycerin at 20°C. Compute the sublayer thickness in inches and the pumping horsepower required at 75 percent efficiency if $L = 40$ ft.

Solution: For glycerin at 20°C, take $\rho = 2.44$ slug/ft^3 and $\mu = 0.0311$ slug/ft·s. Then

$$\mathrm{Re} = \frac{\rho V d}{\mu} = \frac{2.44(60)(1\ \mathrm{ft})}{0.0311} = 4710 \ \ \text{(barely turbulent!)} \quad \text{Smooth: } f_{\mathrm{Moody}} \approx 0.0380$$

$$\text{Then} \quad u^* = V(f/8)^{1/2} = 60\left(\frac{0.0380}{8}\right)^{1/2} \approx 4.13\ \frac{\mathrm{ft}}{\mathrm{s}}$$

The sublayer thickness is defined by $y^+ \approx 5.0 = \rho y u^*/\mu$. Thus

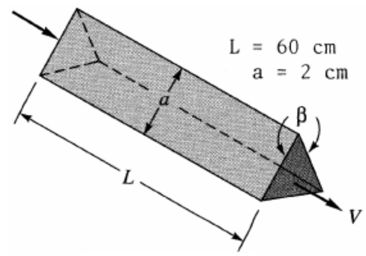
$$y_{\mathrm{sublayer}} \approx \frac{5\mu}{\rho u^*} = \frac{5(0.0311)}{(2.44)(4.13)} = 0.0154\ \mathrm{ft} \approx \mathbf{0.185\ inches} \quad \textit{Ans.}$$

With f known, the head loss and the power required can be computed:

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0380)\left(\frac{40}{1}\right)\frac{(60)^2}{2(32.2)} \approx 85\ \mathrm{ft}$$

$$P = \frac{\rho g Q h_f}{\eta} = \frac{1}{0.75}(2.44)(32.2)\left[\frac{\pi}{4}(1)^2(60)\right](85) = 419000 \div 550 \approx \mathbf{760\ hp} \quad \textit{Ans.}$$

2) Heat exchangers often consist of many triangular passages. Typical is Fig. P6.91, with L = 60 cm and an isosceles-triangle cross section of side length a = 2 cm and included angle β = 80°. If the average velocity is V = 2 m/s and the fluid is SAE 10 oil at 20°C, estimate the pressure drop.



Solution: For SAE 10 oil, take ρ = 870 kg/m^3 and μ = 0.104 kg/m·s. The Reynolds number based on side length a is Re = $\rho Va/\mu$ ≈ **335**, so the flow is *laminar*. The bottom side of the triangle is 2(2 cm)sin40° ≈ *2.57* cm. Calculate hydraulic diameter:

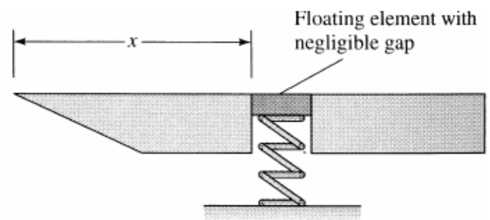
$$A = \frac{1}{2}(2.57)(2\cos 40°) \approx 1.97\ \text{cm}^2; \quad \text{P} = 6.57\ \text{cm}; \quad \text{D}_\text{h} = \frac{4\text{A}}{\text{P}} \approx 1.20\ \text{cm}$$

$$\text{Re}_{\text{D}_\text{h}} = \frac{\rho \text{VD}_\text{h}}{\mu} = \frac{870(2.0)(0.0120)}{0.104} \approx 201; \quad \text{from Table 6.4, } \theta = 40°, \quad \text{fRe} \approx 52.9$$

$$\text{Then} \quad \text{f} = \frac{52.9}{201} \approx 0.263, \quad \Delta\text{p} = \text{f}\frac{\text{L}}{\text{D}_\text{h}}\frac{\rho}{2}\text{V}^2 = (0.263)\left(\frac{0.6}{0.012}\right)\left(\frac{870}{2}\right)(2)^2$$

$$\approx \mathbf{23000\ Pa} \quad \textit{Ans.}$$

3) In the flow of air at 20°C and 1 atm past a flat plate in Fig. P7.43, the wall shear is to be determined at position *x* by a *floating element* (a small area connected to a strain-gage force measurement). At *x* = 2 m, the element indicates a shear stress of 2.1 Pa. Assuming turbulent flow from the leading edge, estimate (a) the stream velocity *U*, (b) the boundary layer thickness δ at the element, and (c) the boundary-layer velocity *u*, in m/s, at 5 cm above the element.



Solution: For air at 20°C, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. The shear stress is

$$\tau_w = 2.1\ \text{Pa} = C_f \frac{\rho}{2}U^2 = \frac{0.027}{(\rho Ux/\mu)^{1/7}}\left(\frac{\rho U^2}{2}\right) = \frac{0.027}{[1.2U(2)/1.8E\text{–}5]^{1/7}}\left(\frac{1.2U^2}{2}\right)$$

Solve for $U \approx \mathbf{34\ \frac{m}{s}}$ *Ans.* (a) Check $Re_x \approx 4.54E6$ (OK, turbulent)

With the local Reynolds number known, solve for local thickness:

$$\delta \approx \frac{0.16x}{Re_x^{1/7}} = \frac{0.16(2\ \text{m})}{(4.54E6)^{1/7}} \approx 0.036\ \text{m} \approx \mathbf{36\ mm} \quad \textit{Ans. (b)}$$

Normally, the log-law, Eq. (7.34), is probably best for estimating the velocity at y = 5 cm above the element. However, from Ans. (*b*) just above, we see that this point is outside the boundary layer. Therefore, the velocity must be **u = U ≈ 34 m/s**. *Ans.* (c).
[NOTE: Part (c) was supposed to state y = 5 **mm**, in which case the correct answer would have been u ≈ *26.5* m/s.]

4) A 2-in.-diameter sphere weighing 0.14 lb is suspended by the jet of air shown in Fig. P9.69 and Video V3.1. The drag coefficient for the sphere is 0.5. Determine the reading on the pressure gage if friction and gravity effects can be neglected for the flow between the pressure gage and the nozzle exit.

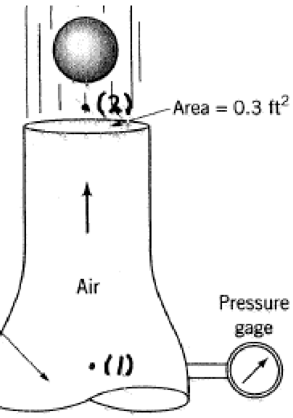
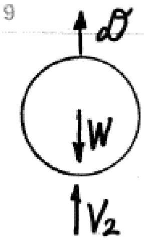


FIGURE P9.69



For equilibrium, $\mathscr{D} = W$ or

$C_D \frac{1}{2}\rho V_2^2 A = W$, where $A = \frac{\pi}{4}D^2$

Thus,

$$V_2 = \left[\frac{2W}{C_D \rho \pi D^2/4}\right]^{1/2}$$

$$= \left[\frac{8\,(0.14\,lb)}{0.5\left(0.00238\,\frac{slugs}{ft^3}\right)\pi\left(\frac{2}{12}\,ft\right)^2}\right]^{1/2} = 104\,\frac{ft}{s}$$

Also,

$$V_1 A_1 = V_2 A_2 \quad \text{or} \quad V_1 = V_2\,\frac{A_2}{A_1} = \left(104\,\frac{ft}{s}\right)\frac{0.3\,ft^2}{0.6\,ft^2} = 52.0\,\frac{ft}{s}$$

and

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{where } p_2 = 0$$

Thus,

$$p_1 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(104\,\frac{ft}{s}\right)^2 - \left(52.0\,\frac{ft}{s}\right)^2\right]$$

$$= 9.65\,\frac{lb}{ft^2}$$

5) Consider water at 20°C flowing past a 1-m-diameter cylinder. a) What doublet strength in m^2/s is required to simulate this flow? b) If the stream pressure is 200 kPa, use inviscid theory to estimate the surface pressure at 135°. If circulation K is added to the cylinder flow, (c) for what value of K will the flow begin to cavitate at the surface? (d) Where on the surface will cavitation begin? (e) For this condition, where will the stagnation points lie? (For water at 20°C, take vapor pressure=2337 Pa)

Solution: For water at 20°C, take $\rho = 998\ \text{kg/m}^3$. The required doublet strength is

$$\lambda = U_\infty a^2 \quad \approx \mathbf{1.25}\ \frac{\mathbf{m^3}}{\mathbf{s}} \qquad \textit{Ans.}$$

The surface pressures are computed from Bernoulli's equation, with Vsurface = $2U_\infty \sin\theta$:

$$p_s + \frac{\rho}{2}(2U_\infty \sin\theta)^2 = p_\infty + \frac{\rho}{2}U_\infty^2, \qquad \text{(b) at } 135°,\ \mathbf{187526\ pa}$$

Cavitation will occur at the lowest pressure point, which is **at the bottom shoulder (θ = 270°)** in Fig. 8.10.

(a) Use Bernoulli's equation to estimate the velocity at θ = 270° if the pressure there is p_{vap}:

$$p_o = 218000\ \text{Pa} = p_{vap} + \frac{\rho}{2}V_{surf}^2 = 2337\ \text{Pa} + \frac{998\ \text{kg/m}^3}{2}\left[(-2)(5\ \text{m/s})\sin(270°) + \frac{K}{0.5\ \text{m}}\right]^2$$

Solve for: $\mathbf{K_{cavitation}} \approx \mathbf{5.26\ m^2/s}$

The locations of the two stagnation points are given by Eq. (8.35):

$$\sin\theta_{stag} = \frac{K}{2U_\infty a} = \mathbf{1.052} \qquad \textbf{No stagnation point on the body}$$