1)

$$P = f(\rho, \mu, D, \Omega, Q)$$

$P = \{FLT^{-1}\}$ $\rho = \{FT^2L^{-4}\}$ $D = \{L\}$ $\mu = \{FTL^{-2}\}$ $\Omega = \{T^{-1}\}$

$Q = \{L^3T^{-1}\}$

$n = 6 \quad j = 3 \Rightarrow n - j = 3$

$\pi_1 = P\Omega^a\rho^bD^c \Rightarrow a = -3 \quad b = -1 \quad c = -5 \Rightarrow \pi_1 = \dfrac{P}{\rho\Omega^3D^5}$

$\pi_2 = Q\Omega^a\rho^bD^c \Rightarrow a = -1 \quad b = 0 \quad c = -3 \Rightarrow \pi_2 = \dfrac{Q}{\Omega D^3}$

$\pi_3 = \mu\Omega^a\rho^bD^c \Rightarrow a = -1 \quad b = -1 \quad c = -2 \Rightarrow \pi_3 = \dfrac{\mu}{\rho\Omega D^2} = Re$

a) $\left.\dfrac{Q}{\Omega D^3}\right|_m = \left.\dfrac{Q}{\Omega D^3}\right|_p \Rightarrow Q_m = \dfrac{\Omega_m D_m^3}{\Omega_p D_p^3} Q_p = \dfrac{1500}{600} \times \dfrac{1^3}{2^3} \times 8 = 2.5\ ft^3/s$

b) $\left.\dfrac{P}{\rho\Omega^3D^5}\right|_m = \left.\dfrac{P}{\rho\Omega^3D^5}\right|_p \Rightarrow P_p = \dfrac{\rho_p}{\rho_m}\dfrac{\Omega_p^3}{\Omega_m^3}\dfrac{D_p^5}{D_m^5} P_m = \dfrac{1000}{1.2}\dfrac{600^3}{1500^3}\dfrac{2^5}{1^5} \times 0.082$

$= 140\ hp$

2)

a) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad v=-V_0 \Rightarrow \frac{\partial u}{\partial x}+0=0 \Rightarrow \frac{\partial u}{\partial x}=0$

$\Rightarrow u=f(y)$

b) $x \quad \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\rho g_x$

$\frac{\partial u}{\partial t}=0$ steady

$\frac{\partial u}{\partial x}=0$ (by cont.)

$v=-V_0$

$\frac{\partial p}{\partial x}=0$ (assumption of problem)

$g_x=0$

$\Rightarrow \quad -\rho V_0\frac{\partial u}{\partial y}=\mu\frac{\partial^2 u}{\partial y^2} \Rightarrow \mu\frac{\partial^2 u}{\partial y^2}+\rho V_0\frac{\partial u}{\partial y}=0$

$\mu\lambda^2+\rho V_0\lambda=0 \Rightarrow \lambda_1=0 \quad \lambda_2=-\frac{\rho V_0}{\mu}$

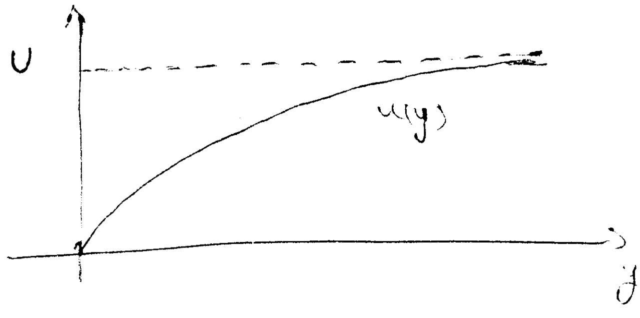
$\Rightarrow u=c_1 e^{0\times y}+c_2 e^{-\frac{\rho V_0}{\mu}y}$

$\Rightarrow u=c_1+c_2 e^{-\frac{\rho V_0}{\mu}y}$

B.C.: $\begin{cases} y=0 \quad u=0 \Rightarrow c_1+c_2=0 \\ y=\infty \quad u=U \Rightarrow c_1=U \end{cases} \Rightarrow c_1=U \quad c_2=-U$

$\Rightarrow u=U\left(1-e^{-\frac{\rho V_0}{\mu}y}\right)$

c)



3)

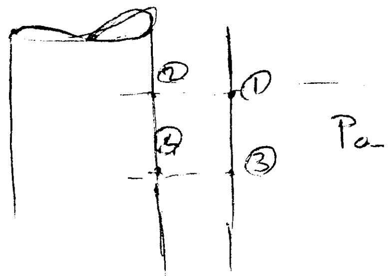
a)

Cont: $\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z} v_z = 0$

$v_r = v_\theta = 0 \implies \frac{\partial v_z}{\partial z} = 0 \implies v_z \neq f(z)$

Also, by axisymmetric $v_z \neq f(\theta) \implies v_z \neq f(z, \theta) \implies v_z = f(r)$

b)



No flow in r direction: $v_r = 0 \implies$ By N-S in r direction: $\frac{dP}{dr} = 0$

$\frac{dP}{dr} = 0 \implies \begin{cases} P_1 = P_2 & (1) \\ P_3 = P_4 & (2) \end{cases}$

Also, $P_1 = P_3 = P_a$ (3)

(1), (2), (3) $\implies P_2 = P_4 \implies \frac{dP}{dz} = 0$

c) N-S in z direction:

$$\cancel{\frac{\partial v_z}{\partial t}} + (v \cdot \nabla) v_z = -\frac{1}{\rho}\cancel{\frac{\partial P}{\partial z}} + g_z + \nu \nabla^2 v_z$$

$$(v \cdot \nabla) v_z = \cancel{v_r \frac{\partial v_z}{\partial r}} + \cancel{\frac{1}{r} v_\theta \frac{\partial v_z}{\partial \theta}} + \cancel{v_z \frac{\partial v_z}{\partial z}}$$

$$\nabla^2 v_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}$$

$$\implies \nu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) + g = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = \frac{-gr}{\nu}$$

$$\Rightarrow \quad r\frac{\partial v_z}{\partial r} = -\frac{gr^2}{2\nu} + c_1$$

$$\Rightarrow \quad \frac{\partial v_z}{\partial r} = \frac{-gr}{2\nu} + \frac{c_1}{r}$$

$$\Rightarrow \quad v_z = -\frac{gr^2}{4\nu} + c_1 \ln r + c_2$$

$$B\,C_s: \begin{cases} r=a & v_z=0 \quad \Rightarrow \quad c_2 = \frac{ga^2}{4\nu} - \frac{gb^2}{2\nu}\ln a \\ r=b & \mu\frac{\partial v_z}{\partial r} = 0 \quad \Rightarrow \quad -\frac{gb}{2\nu} + \frac{c_1}{b} = 0 \Rightarrow c_1 = \frac{gb^2}{2\nu} \end{cases}$$

$$\Rightarrow v_z = -\frac{gr^2}{4\nu} + \frac{gb^2}{2\nu}\ln r + \frac{ga^2}{4\nu} - \frac{gb^2}{2\nu}\ln a$$

$$= \frac{gb^2}{2\nu}\ln\left(\frac{r}{a}\right) - \frac{g}{4\nu}\left(r^2 - a^2\right)$$

d) $$\left.\frac{\partial v_z}{\partial r}\right|_{r=a} = -\frac{ga}{2\nu} + \frac{gb^2}{2a\nu}$$

$$\tau = \mu\left.\frac{\partial v_z}{\partial r}\right|_{r=a} = \mu\left(-\frac{ga}{2\nu} + \frac{gb^2}{2a\nu}\right)$$

$$F = \tau . A = \tau . 2\pi R . L = \mu\left(-\frac{ga}{2\nu} + \frac{gb^2}{2a\nu}\right)2\pi a L$$

$$= \frac{\mu g}{2\nu}\left(\frac{b^2 - a^2}{a}\right)2\pi a L$$

$$= \rho g \pi L\left(b^2 - a^2\right)$$

e) $$Q = \int_a^b 2\pi r v_z \, dr$$

$$= 2\pi\left\{\frac{gb^2}{2\nu}\int_a^b r\ln\frac{r}{a}\,dr - \frac{g}{4\nu}\int_a^b (r^3 - ra^2)\,dr\right\}$$

$$= \frac{\pi g b^2}{\nu}\left[\frac{1}{2} r^2 \ln\frac{r}{a} - \frac{1}{4} r^2\right]_a^b - \frac{\pi g}{2\nu}\left[\frac{r^4}{4} - \frac{r^2 a^2}{2}\right]_a^b$$

$$= \frac{\pi g b^2}{\nu}\left[\frac{1}{2} b^2 \ln\frac{b}{a} - \frac{1}{4} b^2 + \frac{1}{4} a^2\right] - \frac{\pi g}{2\nu}\left[\frac{b^4}{4} - \frac{b^2 a^2}{2} - \frac{a^4}{4} + \frac{a^4}{2}\right.$$

$$= \frac{\pi g}{8\nu}\left[4 b^4 \ln\frac{b}{a} - 3 b^4 + 4 a^2 b^2 - a^4\right]$$