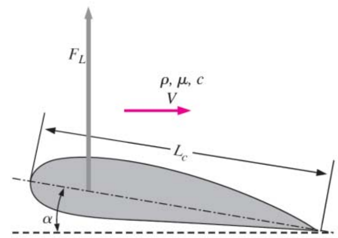
Fluid ID:-----------------	2^{nd} Midterm Exam	Time: 50 minutes
Name: ------------------	Course: 58:160, Fall 2008	

1. Some aeronautical engineers are designing an airplane and wish to predict the lift produced by their new wing design. The chord length L_c of the wing is 1.12 m. The prototype is to fly at V = 52.0 m/s close to the ground where T = 25°C and $\rho=\rho_a=1.2 kg/m^3$. They build a one-tenth scale model of the wing to test in a pressurized wind tunnel. (a) Establish a relationship between lift force F_L ,V, L_c, ρ, μ, c (sound velocity), and α (attack angle). (b) Assume that the air in the wind tunnel has $\rho_{wind\text{-}tunnel}=5\rho_a$ and sound velocity is negligible. At what speed should they run the wind tunnel in order to achieve dynamic similarity? (c) What is the lift force of the prototype if the lift force of the model is 200 N?



2. Consider steady, incompressible, laminar flow of a Newtonian fluid in an infinitely long round pipe of diameter D or radius R = D/2 inclined at angle. There is no applied pressure gradient ($dp/dx = 0$). Instead, the fluid flows down the pipe due to gravity alone. We adopt the coordinate system shown, with x down the axis of the pipe. (a) Write up all assumptions you need to make. (b) List all boundary conditions. (c) Derive an expression for the x-component of velocity u as a function of radius r and the other parameters of the problem. (d) Assume that x-component of velocity has this form: $u=C_1r^2+C_2\ln(r)+C_3$. By using boundary conditions and substituting this form of the solution in the expression derived at part (c) find C_1, C_2, and C_3. *(Note: d(ln(r))/dr=1/r)*

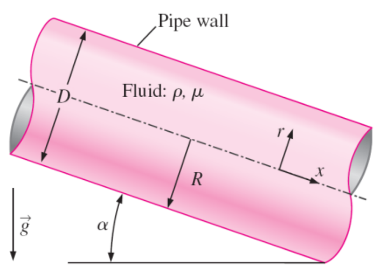


Table of Dimensions:

Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Expansion coefficient	β	Θ^{-1}	Θ^{-1}

Appendix D
Equations of Motion in Cylindrical Coordinates

The equations of motion of an incompressible newtonian fluid with constant μ, k, and c_p are given here in cylindrical coordinates (r, θ, z), which are related to cartesian coordinates (x, y, z) as in Fig. 4.2:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z \tag{D.1}$$

The velocity components are v_r, v_θ, and v_z. The equations are:

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0 \tag{D.2}$$

Convective time derivative:

$$\mathbf{V}\cdot\nabla = v_r\frac{\partial}{\partial r} + \frac{1}{r}v_\theta\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z} \tag{D.3}$$

Laplacian operator:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{D.4}$$

The r-momentum equation:

$$\frac{\partial v_r}{\partial t} + (\mathbf{V}\cdot\nabla)v_r - \frac{1}{r}v_\theta^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r + \nu\left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right) \tag{D.5}$$

The θ-momentum equation:

$$\frac{\partial v_\theta}{\partial t} + (\mathbf{V}\cdot\nabla)v_\theta + \frac{1}{r}v_r v_\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + g_\theta + \nu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right) \tag{D.6}$$

The z-momentum equation:

$$\frac{\partial v_z}{\partial t} + (\mathbf{V}\cdot\nabla)v_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z + \nu\nabla^2 v_z \tag{D.7}$$