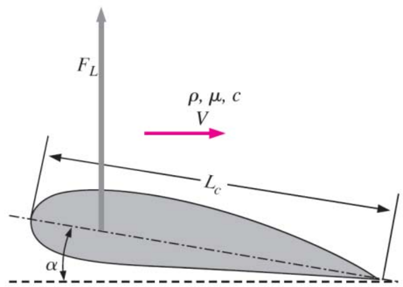
Fluid ID:-----------------	2^{nd} Midterm Exam	Time: 50 minutes
Name: ------------------	Course: 58:160, Fall 2008	

1. Some aeronautical engineers are designing an airplane and wish to predict the lift produced by their new wing design. The chord length L_c of the wing is 1.12 m. The prototype is to fly at V = 52.0 m/s close to the ground where T = 25°C and $\rho=\rho_a=1.2 kg/m^3$. They build a one-tenth scale model of the wing to test in a pressurized wind tunnel. (a) Establish a relationship between lift force F_L ,V, L_c, ρ, μ, c (sound velocity), and α (attack angle). (b) Assume that the air in the wind tunnel has $\rho_{wind\text{-}tunnel}=5\rho_a$ and sound velocity is negligible. At what speed should they run the wind tunnel in order to achieve dynamic similarity? (c) What is the lift force of the prototype if the lift force of the model is 200 N?



Solution:

Total score for this problem= (40)

(a)

Step 1: There are seven variables and constants in this problem; n = 7. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

$$F_L = f\left(V, L_c, \rho, \mu, c, \alpha\right) \quad \underbrace{n = 7}_{3}$$

where F_L is the lift force on the wing, V is the fluid speed, L_c is the chord length, ρ is the fluid density, μ is the fluid viscosity, c is the speed of sound in the fluid, and α is the angle of attack of the wing.

Step 2: The primary dimensions of each parameter are listed:

$$\underbrace{F_L = \{MLT^{-2}\}}_{1}$$

$$\underbrace{V = \{LT^{-1}\}}_{1}$$

$$\underbrace{L_c = \{L\}}_{1}$$

$$\underbrace{\rho = \{ML^{-3}\}}_{1}$$

$$\underbrace{\mu = \{ML^{-1}T^{-1}\}}_{1}$$

$$\underbrace{c = \{LT^{-1}\}}_{1}$$

$$\underbrace{\alpha = \{1\}}_{1}$$

Step 3: As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (M, L, and T).

Reduction (first guess): $\underbrace{j = 3}_{2}$

If this value of j is correct, the expected number of Π 's is $\underbrace{k = n - j = 7 - 3 = 4}_{2}$.

Step 4: We need to choose three repeating parameters since j = 3. We cannot choose both V and c since their dimensions are identical. Nor can we pick α since it is already dimensionless. It would not be desirable to have μ appear in all the Π 's. The best choice of repeating parameters is thus V, L_c, and ρ.

$$\Pi_1 = \underbrace{F_L V^{a_1} L_c^{b_1} \rho^{c_1}}_{2} = \{(MLT^{-2})(LT^{-1})^{a_1}(L)^{b_1}(ML^{-3})^{c_1}\} = \{M^0 L^0 T^0\}$$

$$\Rightarrow a_1 = -2, b_1 = -2, c_1 = -1 \Rightarrow \Pi_1 = \frac{F_L}{\rho V^2 L_c^2} \qquad or\ \Pi_1 = \frac{F_L}{\underbrace{1/2\rho V^2 L_c^2}_{1}} = C_L$$

$$\Pi_2 = \underbrace{\mu V^{a_2} L_c^{b_2} \rho^{c_2}}_{2} = \{(ML^{-1}T^{-1})(LT^{-1})^{a_2}(L)^{b_2}(ML^{-3})^{c_2}\} = \{M^0 L^0 T^0\}$$

$$\Rightarrow a_2 = -1, b_2 = -1, c_2 = -1 \Rightarrow \Pi_2 = \frac{\mu}{\rho V L_c} \qquad or\ \Pi_2 = \frac{\rho V L_c}{\underbrace{\mu}_{1}} = \mathrm{Re}$$

$$\Pi_3 = \underbrace{c V^{a_3} L_c^{b_3} \rho^{c_3}}_{2} = \{(LT^{-1})(LT^{-1})^{a_3}(L)^{b_3}(ML^{-3})^{c_3}\} = \{M^0 L^0 T^0\}$$

$$\Rightarrow a_3 = -1, b_2 = 0, c_2 = 0 \Rightarrow \Pi_3 = \frac{c}{V} \qquad or\ \Pi_3 = \frac{V}{\underbrace{c}_{1}} = Ma$$

Finally, the attack angle is already dimensionless.

$$\Pi_4 = \underbrace{\alpha}_{3}$$

Step 5: We write the final functional relationship.

$$\underbrace{C_L = \frac{F_L}{1/2\rho V^2 L_c^2} = f(\text{Re}, Ma, \alpha)}_{1}$$

(b) Since sound velocity is negligible, we need to only set Re number of the model and prototype to be the same.

$$\text{Re}_m = \text{Re}_p \Rightarrow \underbrace{\frac{\rho_m V_m L_{cm}}{\mu_m} = \frac{\rho_p V_p L_{cp}}{\mu_p}}_{2}$$

$$\left.\begin{array}{l} \underbrace{\rho_m = 5\rho_a}_{1} \\ \underbrace{\rho_p = \rho_a}_{1} \\ \underbrace{\mu_m = \mu_p}_{1} \\ \underbrace{L_{cm} = 1/10 L_{cp}}_{1} \end{array}\right\} \Rightarrow \underbrace{V_m = 2V_p}_{1} = \underbrace{2 \times 52 = 104 m/s}_{1}$$

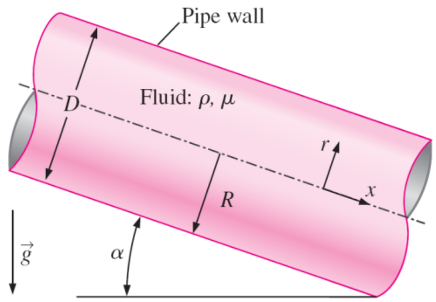
(c)

$$C_{Lm} = C_{Lp} \Rightarrow \underbrace{\frac{F_{Lm}}{1/2\rho_m V_m^2 L_{cm}^2} = \frac{F_{Lp}}{1/2\rho_p V_p^2 L_{cp}^2}}_{2}$$

$$\left.\begin{array}{l} \rho_m = 5\rho_a \\ \rho_p = \rho_a \\ L_{cm} = 1/10 L_{cp} \\ V_m = 2V_p \\ \underbrace{F_{Lm} = 200}_{1} \end{array}\right\} \Rightarrow \underbrace{F_{Lp} = F_{Lm}\left(\frac{L_{cp}}{L_{cm}}\right)^2 \left(\frac{V_p}{V_m}\right)^2 \left(\frac{\rho_p}{\rho_m}\right)^2}_{1} = \underbrace{200 \times 10^2 \times 0.5^2 \times (1/5) = 1000\ N = 1KN}_{1}$$

2. Consider steady, incompressible, laminar flow of a Newtonian fluid in an infinitely long round pipe of diameter D or radius R = D/2 inclined at angle. There is no applied pressure gradient ($dp/dx = 0$). Instead, the fluid flows down the pipe due to gravity alone. We adopt the coordinate system shown, with x down the axis of the pipe. (a) Write up all assumptions you need to make. (b) List all boundary conditions. (c) Derive an expression for the x-component of velocity u as a function of radius r and the other parameters of the problem. (d) Assume that x-component of velocity has this form: $u = C_1 r^2 + C_2 \ln(r) + C_3$. By

using boundary conditions and substituting this form of the solution in the expression derived at part (c) find C_1, C_2, and C_3. *(Note: d(ln(r))/dr=1/r)*



Solution:

Total score for this problem= (40)

(a)

We are to calculate u(r) for flow inside an inclined round pipe. Assumptions We number and list the assumptions for clarity:

1. The flow is steady, i.e. any time derivative is zero. (1)
2. This is a parallel flow (the r component of velocity, u_r, is zero). (1)
3. The pressure is constant everywhere except for hydrostatic pressure. (1)
4. The velocity field is axisymmetric with no swirl, implying that $u_\theta = 0$ and all derivatives with respect to θ are zero. (1)

(b)

The first boundary condition comes from imposing the no slip condition at the pipe wall: 1. at r = R, u=0. (2)

The second boundary condition comes from the fact that the centerline of the pipe is an axis of symmetry: 2. at r = 0, du/dr = 0. (2)

(c)

Continuity:
$$\underbrace{\underbrace{\frac{1}{r}\frac{\partial(ru_r)}{\partial r}}_{=0;\,assumption\,2}}_{2} + \underbrace{\underbrace{\frac{1}{r}\frac{\partial(ru_\theta)}{\partial \theta}}_{=0;\,assumption\,4}}_{2} + \frac{\partial u}{\partial x} = 0 \quad or \quad \underbrace{\frac{\partial u}{\partial x}}_{2} = 0$$

Continuity tells us that u is not a function of x. Furthermore, since u is not a function of time (Assumption1) or θ (Assumption 4), we conclude that u is at most a function of r, u=u(r).

We now simplify the x momentum equation as far as possible:

x-momentum:

$$\rho\left(\underbrace{\underbrace{\frac{\partial u}{\partial t}}_{=0;\,assumption1}}_{2}+\underbrace{\underbrace{u_r\frac{\partial u}{\partial r}}_{=0;\,assumption2}}_{2}+\underbrace{\underbrace{\frac{u_\theta}{r}\frac{\partial u}{\partial\theta}}_{=0;\,assumption4}}_{2}+\underbrace{\underbrace{u\frac{\partial u}{\partial x}}_{=0;\,continuity}}_{2}\right)=\underbrace{\underbrace{-\frac{\partial p}{\partial x}}_{=0;\,assumption3}}_{2}+\underbrace{\underbrace{\rho g_x}_{\rho g\sin\alpha}}_{2}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\underbrace{\underbrace{\frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2}}_{=0;\,assumption4}}_{2}+\underbrace{\underbrace{\frac{\partial^2 u}{\partial x^2}}_{continuity}}_{2}\right)$$

or:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)=\frac{-\rho g\sin\alpha}{\mu}}_{2}$$

(d) $u=C_1r^2+C_2\ln r+C_3$

x-momentum:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)=\frac{-\rho g\sin\alpha}{\mu}\Rightarrow\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(2C_1r^2+C_2\right)=\frac{-\rho g\sin\alpha}{\mu}}_{1}\Rightarrow 4C_1=\frac{-\rho g\sin\alpha}{\mu}\Rightarrow\underbrace{C_1=\frac{-\rho g\sin\alpha}{4\mu}}_{1}$$

Boundary conditions:

Boundary condition (2):

$$\left.\frac{du}{dr}\right|_{r=0}=0\Rightarrow\underbrace{0+\frac{C_2}{0}=0}_{1}\Rightarrow\underbrace{C_2=0}_{1}$$

Boundary condition (1):

$$u(r=R)=0\Rightarrow\underbrace{\frac{-\rho g\sin\alpha}{4\mu}R^2+0+C_3=0}_{1}\Rightarrow\underbrace{C_3=\frac{\rho g\sin\alpha}{4\mu}R^2}_{1}$$

Therefore:

$$\underbrace{u=\frac{\rho g\sin\alpha}{4\mu}(R^2-r^2)}_{2}$$