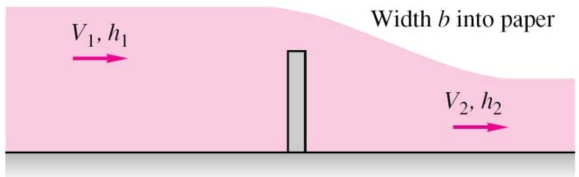
Fluid ID:-----------------	**1st Midterm Exam**	**Time: 50 minutes**
Name: ------------------	**Course: 58:160, Fall 2008**	

1. A river of width b and depth h_1 passes over a submerged obstacle, or "drowned weir," as shown, emerging at a new flow condition (V_2, h_2). Neglect atmospheric pressure, and assume that the water pressure is hydrostatic at both sections 1 and 2. (a) Derive an expression for the force exerted by the river on the obstacle in terms of V_1, h_1, h_2, b, ρ, and g. Neglect water friction on the river bottom. (b) Find head loss caused by the obstacle in terms of V_1, h_1, h_2, b, ρ, and g. (c) Find h_1 for which head loss is a maximum.



Solution:

Total score for this problem= (40)

$$\underbrace{V_1 h_1}_{2} = \underbrace{V_2 h_2}_{2} \Rightarrow \underbrace{V_2 = V_1 h_1 / h_2}_{1} \quad (5)$$

$$\sum F_x = \underbrace{-F + F_{p1} - F_{p2}}_{2+2+2} = \underbrace{\dot{m}(V_2 - V_1)}_{2+2} \quad (10)$$

$$F_{p1} = P_c A = \underbrace{1/2 \rho g h_1}_{2} \underbrace{(h_1 b)}_{1} \quad (3)$$

$$F_{p2} = P_c A = \underbrace{1/2 \rho g h_2}_{2} \underbrace{(h_2 b)}_{1} \quad (3)$$

$$\dot{m} = \rho h_1 b V_1 \quad (2)$$

$$\Rightarrow \underbrace{F = \frac{1}{2}\rho g b (h_1^2 - h_2^2) - \rho h_1 b V_1^2 (\frac{h_1}{h_2} - 1)}_{1} \quad (1)$$

$$\underbrace{\frac{p_1}{\gamma}}_{1.5} + \underbrace{\frac{V_1^2}{2g}}_{2} + \underbrace{h_1}_{1} = \underbrace{\frac{p_2}{\gamma}}_{1.5} + \underbrace{\frac{V_2^2}{2g}}_{2} + \underbrace{h_2}_{1} + \underbrace{h_l}_{1} \quad (10)$$

$$\underbrace{p_1 = 0;}_{1} \underbrace{p_2 = 0}_{1} \quad (2)$$

$$V_2 = V_1 h_1 / h_2$$

$$\Rightarrow \underbrace{h_l = h_1 - h_2 + \frac{V_1^2}{2g}(1 - (\frac{h_1}{h_2})^2)}_{1} \quad (1)$$

$$\underbrace{\frac{dh_l}{dh_1} = 0}_{2} \quad \Rightarrow 1 - \frac{V_1^2}{g h_2^2} h_1 = 0 \Rightarrow \underbrace{h_1 = \frac{g h_2^2}{V_1^2}}_{1} \quad (3)$$

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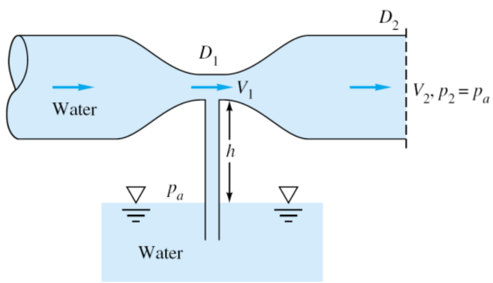
2. A necked-down section in a pipe flow, called a *venturi*, develops a low throat pressure which can aspirate fluid upward from a reservoir, as shown. Assuming no losses, derive an expression in terms of D_1, D_2, h, and g for the velocity V_1 which is just sufficient to bring reservoir fluid into the throat.



Solution:

Total score for this problem= (40)

$$\underbrace{\frac{p_1}{\gamma}}_{1.5}+\underbrace{\frac{V_1^2}{2g}}_{2}+\underbrace{z_1}_{1.5}=\underbrace{\frac{p_2}{\gamma}}_{1.5}+\underbrace{\frac{V_2^2}{2g}}_{2}+\underbrace{z_2}_{1.5}\ (10)$$

$$z_1 = z_2\ (2)$$

$$p_2 = p_a\ (2)$$

$$\underbrace{V_1A_1}_{5}=\underbrace{V_2A_2}_{5}\ (10)$$

$$\underbrace{A_1=\pi D_1^2/4}_{1.5};\ \underbrace{A_2=\pi D_2^2/4}_{1.5}\Rightarrow \underbrace{V_2=V_1D_1^2/D_2^2}_{1}\ \ (4)$$

$$\underbrace{p_a}_{3}=\underbrace{p_1}_{3}+\underbrace{\rho gh}_{3}\Rightarrow \underbrace{p_a-p_1=\rho gh}_{1}\ (10)$$

$$\Rightarrow \underbrace{\frac{V_1^2}{2g}(1-D_1^4/D_2^4)=h}_{1}\Rightarrow \underbrace{V_1=\sqrt{\frac{2gh}{1-D_1^4/D_2^4}}}_{1}\ (2)$$

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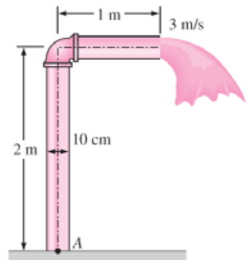
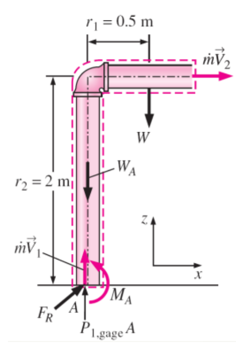
3. Underground water is pumped to a sufficient height through a 10-cm diameter pipe that consists of a 2-m-long vertical and 1-m-long horizontal section, as shown. Water discharges to atmospheric air at an average velocity of 3 m/s, and the mass of the horizontal pipe section when filled with water is 12 kg per meter length. The pipe is anchored on the ground by a concrete base. Determine (a) the bending moment acting at the base of the pipe (point A) and (b) the required length of the horizontal section that would make the moment at point A zero.



Solution:

Total score for this problem= (40)



$$\underbrace{\sum M}_{2} = \underbrace{\sum\nolimits_2 r\dot{m}V}_{2} - \underbrace{\sum\nolimits_1 r\dot{m}V}_{2} \quad (6)$$

$$LHS = \underbrace{M_A}_{2} - \underbrace{r_1 W}_{2} \quad (4)$$

$$RHS = \sum\nolimits_2 r\dot{m}V - \sum\nolimits_1 r\dot{m}V = \underbrace{-r_2\dot{m}_2V_2}_{2} - \underbrace{0}_{2} \quad (4)$$

$$\underbrace{M_A = r_1W - r_2\dot{m}_2V}_{1} \quad (1)$$

(a)

$$\left.\begin{aligned}
&\underbrace{\dot m_1 = \dot m_2 = \rho A_2 V_2}_{3} = 1000\times\underbrace{(\pi(0.1)^2/4)}_{1}\times 3 = \underbrace{23.56 kg/s}_{1}\ (5)\\
&\underbrace{W = mg}_{4} = 12\times1\times9.81 = \underbrace{118N}_{1}\ (5)\\
&r_1 = 0.5\ \ (2)\\
&r_2 = 2\ \ (2)
\end{aligned}\right\} \Rightarrow M_A = 0.5\times118 - 2\times23.56\times3 = \underbrace{-82.5 N.m}_{1}\ \ (1)$$

Therefore, a moment of 82.5 N · m acts at the stem of the pipe in the clockwise direction.

(b)

$$\left.\begin{aligned}
&\underbrace{M_A = r_1 W - r_2 \dot m_2 V}_{3} = \underbrace{0}_{2}\ (5)\\
&r_1 = \frac{L}{2}\ (2)\\
&W = \underbrace{12\times L\times 9.81}_{1} = \underbrace{118L}_{1}\ \ (2)\\
&r_2 \dot m_2 V = 2\times 23.56\times 3 = 141.4
\end{aligned}\right\} \Rightarrow \underbrace{L = \sqrt{\frac{2\times141.4}{118}} = 1.55m}_{1}\ (1)$$