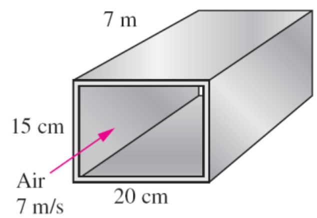
Fluid ID:-----------------	Final Exam	Time: 100 minutes
Name: ------------------	Course: 58:160, Fall 2008	

1. Air enters a 7-m-long section of a rectangular duct of cross section 15 cm x 20 cm made of commercial steel (ε=0.045mm) at 1atm and 35°C at an average velocity of 7 m/s. Disregarding the entrance effects, determine the fan power needed to overcome the pressure losses in this section of the duct. (Air at 35^0C : ρ_{air}= 1.145 kg/m^3, ν_{air}= 1.655E-5 m^2/s)



$$D_h = \frac{4A}{P} = \frac{4ab}{2(a+b)} = \frac{4(0.15)(0.2)}{2(0.15+0.2)} = 0.1714\ m$$

$$Q = AV = (a\times b)V = (0.15\times 0.2)(7) = 0.21\ m^3/s$$

$$\mathrm{Re} = \frac{VD_h}{\upsilon} = 72490$$ Re>4000 -> Turbulent

$$\frac{\varepsilon}{D_h} = \frac{0.000045}{0.1714} = 2.625\times 10^{-4}$$

Moody chart: f=0.0203

$$h = f\frac{L}{D}\frac{V^2}{2g} = 0.02034\frac{7}{0.1714}\frac{7^2}{2\times 9.81} = 2.07\ m$$
$$P = \rho g h Q = (1.145)(9.81)(2.07)(0.21) = 4.9W$$

2. Air at 20^0C flows through a 14-cm-diameter smooth tube under fully developed conditions. The shear stress on the wall is τ_w=0.062 Pa. Find: (a) Friction velocity. (b) Velocity at the edge of inner layer. (c) Centerline velocity (V at r=0). Assume that the pipe roughness is not negligible and it is 0.005 m, (d) what would be centerline velocity for the rough pipe? (Air at 20^0C : ρ_{air}= 1.205 kg/m^3, ν_{air}= 1.51E-5 m^2/s)

(a) $\tau_w = \rho u*^2 \Rightarrow u* = \left(\frac{\tau_w}{\rho}\right)^{0.5} = \left(\frac{0.062}{1.205}\right)^{0.5} = 0.227\ m/s$

(b) $u^+ = y^+ = 5 \Rightarrow \frac{u}{u*} = 5 \Rightarrow u = 5u* = 1.135\ m/s$

(c)
$$\frac{u(r)}{u*} = \frac{1}{\kappa}\ln\left(\frac{(R-r)u*}{\upsilon}\right) + B \Rightarrow u_C = 0.227\left(\frac{1}{0.41}\ln\left(\frac{(0.14/2-0)0.227}{1.51E-5}\right)+5\right) = 4.98\ m/s$$

(d) $\varepsilon^{+} = \dfrac{\varepsilon u*}{\upsilon} = \dfrac{0.005(0.227)}{1.51E-5} = 75.16 \Rightarrow \textit{fully rough flow}$

For fully rough flow:

$$\frac{u}{u*} = \frac{1}{\kappa}\ln\left(\frac{yu*}{\upsilon}\right) + B - \Delta B = \frac{1}{0.41}\ln\left(\frac{yu*}{\upsilon}\right) + 5 - \left(\frac{1}{0.41}\ln\left(\frac{\varepsilon u*}{\upsilon}\right) - 3.5\right) = \frac{1}{0.41}\ln\left(\frac{y}{\varepsilon}\right) + 8.5$$

$$\frac{u(r)}{u*} = \frac{1}{0.41}\ln\left(\frac{(R-r)}{\varepsilon}\right) + 8.5 \Rightarrow u_C = 0.227\left(\frac{1}{0.41}\ln\left(\frac{(0.14/2-0)}{0.005}\right) + 8.5\right) = 3.39\ m/s$$

3. A boundary layer develops along the walls of a rectangular wind tunnel. The air is at 20°C and atmospheric pressure. The boundary layer starts upstream of the contraction and grows into the test section, as shown in Fig. a. By the time it reaches the test section, the boundary layer is fully turbulent. The boundary layer profile and its thickness are measured at both the beginning ($x = x_1$) and the end ($x = x_2$) of the bottom wall of the wind tunnel test section, as shown in Fig. b. The test section is 1.8 m long and 0.50 m wide (into the page in figure) and wind tunnel velocity is V=10 m/s. The following measurements are made:

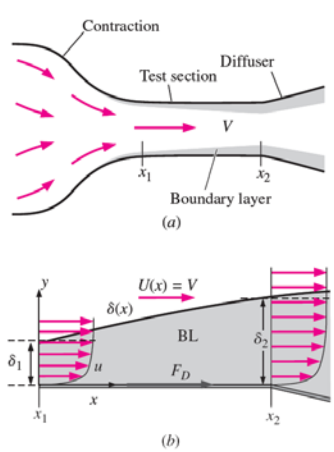
$$\delta_1 = 4.2\,cm \quad \delta_2 = 7.7\,cm$$

At both locations the boundary layer profile fits better to a one-eighth-power law approximation:

$$\frac{u}{U} \cong \left(\frac{y}{\delta}\right)^{1/8} \quad y \le \delta$$

$$\frac{u}{U} = 1 \quad y \ge \delta$$

Estimate (a) momentum thickness in term of boundary layer thickness δ and (b) the total skin friction drag force F_D acting on the bottom wall of the wind tunnel test section. (Air at 20^0C : ρ_{air}= 1.205 kg/m^3, ν_{air}= 1.51E-5 m^2/s)



$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^{\delta}\left(\frac{y}{\delta}\right)^{1/8}\left(1 - \left(\frac{y}{\delta}\right)^{1/8}\right)dy$$

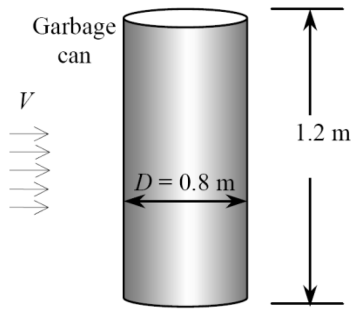
$$\Rightarrow \theta = \int_0^{\delta}\left(\frac{y}{\delta}\right)^{1/8}dy - \int_0^{\delta}\left(\frac{y}{\delta}\right)^{2/8}dy = \frac{8}{9\delta^{1/8}}y^{9/8}\Big|_0^{\delta} - \frac{4}{5\delta^{1/4}}y^{5/4}\Big|_0^{\delta} = \left(\frac{8}{9}\delta - \frac{4}{5}\delta\right) = \frac{4}{45}\delta$$

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

$$F_D = \int_{x_1}^{x_2} \tau_w dA = \int_{x_1}^{x_2} \rho U^2 \frac{d\theta}{dx} b dx = \rho U^2 b \int_{x_1}^{x_2} \frac{d\theta}{dx} dx = \rho U^2 b(\theta_2 - \theta_1) = \rho U^2 b \frac{4}{45}(\delta_2 - \delta_1)$$

$$\Rightarrow F_D = 1.205(10^2)(0.5)\frac{4}{45}(7.7 - 4.2) = 0.19\ N$$

4. A 0.80-m-diameter, 1.2-m-high garbage can is found in the morning tipped over due to high winds during the night. Assuming the average density of the garbage inside to be 150 kg/m^3 and taking the air density to be 1.25 kg/m^3, estimate the wind velocity during the night when the can was tipped over. Take the drag coefficient of the can to be 0.7.

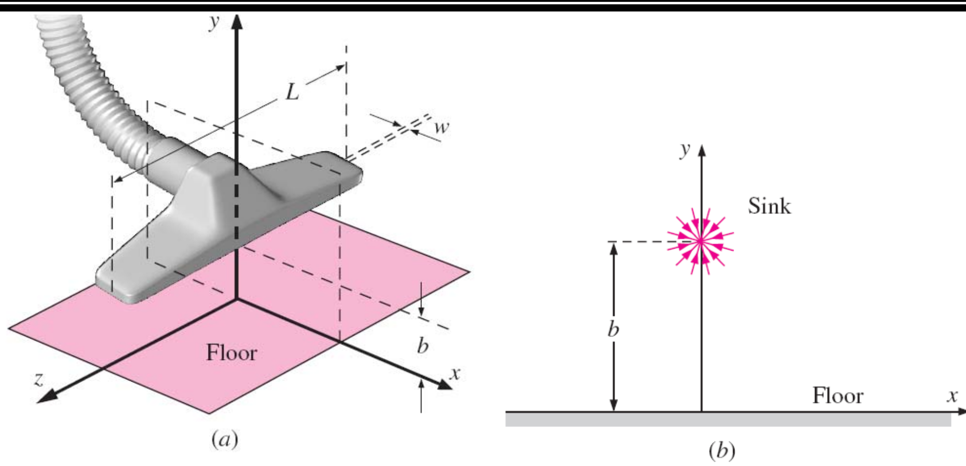


$$W = \rho g V = \rho g \frac{\pi D^2}{4} h = (150)(9.81)(\frac{\pi 0.8^2}{4})(1.2) = 887.6\,N$$

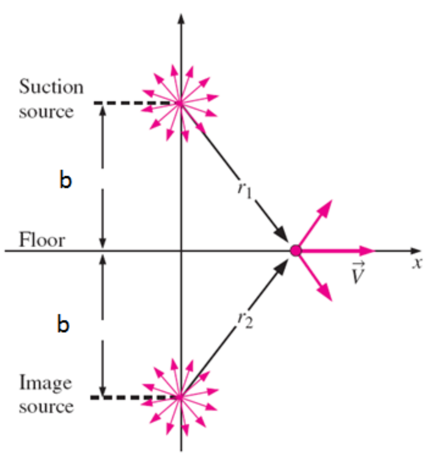
$$\sum M_{contact} = 0 \Rightarrow F_D \times (h/2) - W \times (D/2) = 0 \Rightarrow F_D = \frac{WD}{h} = 591.7\,N$$

$$F_D = C_D A \frac{\rho V^2}{2} \Rightarrow 591.7 = 0.7(1.2 \times 0.8)\frac{1.25 V^2}{2} \Rightarrow V = 37.5\ m/s$$

5. Consider the flow of air into the floor attachment nozzle of a typical household vacuum cleaner. The width of the nozzle inlet slot is w= 2.0 mm, and its length is L = 35.0 cm. The slot is held a distance b= 2.0 cm above the floor, as shown in Fig a. The total volume flow rate through the vacuum hose is Q= 0.110 m^3/s. Consider the flow field in the center plane of the attachment (the xy-plane in Fig) as a 2D sink line flow near a wall, as shown in Fig b. The strength of the sink (m) can be estimated by m=(Q/L)/(2π). Calculate (a) the velocity and (b) gage pressure distribution along the x axis. (c) What is the maximum magnitude of speed along the floor, and where does it occur?



We can consider sinks as sources with negative source strength. Then:



$$V_x = 2V_{source}\cos\theta = 2\frac{-m}{r_1}\frac{x}{r_1} = -2m\frac{x}{r_1^2} = -2m\frac{x}{x^2+b^2} = -2\left(\frac{Q}{2\pi L}\right)\frac{x}{x^2+b^2} = -\frac{Q}{\pi L}\left(\frac{x}{x^2+b^2}\right)$$

$$P_\infty + \frac{\rho V_\infty^2}{2} = P + \frac{\rho V_x^2}{2} \Rightarrow P_{gage} = P - P_\infty = \frac{\rho}{2}\left(V_\infty^2 - V_x^2\right) = \frac{\rho}{2}\left(V_\infty^2 - \frac{Q^2}{\pi^2 L^2}\left(\frac{x}{x^2+b^2}\right)^2\right)$$

Max V_x:

$$\frac{dV_x}{dx} = 0 \Rightarrow \frac{d}{dx}\left(\frac{x}{x^2+b^2}\right) = 0 \Rightarrow \frac{(1)(x^2+b^2)-(x)(2x)}{(x^2+b^2)^2} = 0 \Rightarrow x^2+b^2-2x^2 = 0 \Rightarrow x = \pm b = \pm 0.02m$$

$$V_x(x=\pm b) = -\frac{Q}{\pi L}\left(\frac{\pm b}{b^2+b^2}\right) = \mp\frac{Q}{2\pi L b} = \mp\frac{0.110}{2\pi(0.35)(0.02)} = \mp 2.5\ m/s$$