# V&V Procedures for CFDLabs

## 58:160 Intermediate Mechanics of Fluids

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The purpose of this document is to summarize and demonstrate V&V procedures to be used in CFDLab reports.

**Nomenclature:**

$S_{g1}$: solution from fine grid

$S_{g2}$: solution from medium grid

$S_{g3}$: solution from coarse grid

$R_g$: grid convergence ratio

$r_g$: grid refinement ratio (1.414 for CFDLab1)

$P_g$: order of accuracy for grid

$P_{gest}$: theoretical order of accuracy, 2 for 2nd order and 1 for 1st order schemes

$\delta^*_{REg1}$: grid error from Richardson Extrapolation based on fine mesh solution

$C_g$: correction factor for grid

$U_g$: grid uncertainty based on the absolute value of the corrected error estimate plus the amount of the correction.

$U_{gc}$: grid uncertainty based on the absolute value of the amount of the correction.

**Formulae:**

$\varepsilon_{g21} = S_{g2} - S_{g1}$

$\varepsilon_{g32} = S_{g3} - S_{g2}$

$R_g = \frac{\varepsilon_{g21}}{\varepsilon_{g32}}$

If monotonically converged ($0<R_g<1$), then:

$$P_g = \frac{\ln\left(\frac{\varepsilon_{g32}}{\varepsilon_{g21}}\right)}{\ln(r_g)}$$

$$\delta^*_{REg1} = \frac{\varepsilon_{g21}}{(r_g^{P_g} - 1)}$$

$$Cg = \frac{r_g^{P_g} - 1}{r_g^{P_{gest}} - 1}$$

$$U_g = \begin{cases} \left(9.6(1-C_g)^2 + 1.1\right)\left|\delta^*_{REg1}\right| & \text{if} \quad \left|1-C_g\right| < 0.125 \\ \left(2\left|1-C_g\right| + 1\right)\left|\delta^*_{REg1}\right| & \text{if} \quad \left|1-C_g\right| \geq 0.125 \end{cases}$$

$$U_{gc} = \begin{cases} \left(2.4(1-C_g)^2 + 0.1\right)\left|\delta^*_{REg1}\right| & \text{if} \quad \left|1-C_g\right| < 0.25 \\ \left(\left|1-C_g\right|\right)\left|\delta^*_{REg1}\right| & \text{if} \quad \left|1-C_g\right| \geq 0.25 \end{cases}$$

Following examples demonstrate grid studies for friction factor of laminar pipe flows.

**Example 1:**

$$\varepsilon_{g21} = S_{g2} - S_{g1} = -0.0027916$$
$$\varepsilon_{g32} = S_{g3} - S_{g2} = -0.0157815$$
$$P_g = \frac{\ln(\frac{\varepsilon_{g32}}{\varepsilon_{g21}})}{\ln(r_g)} = 2.49907$$
$$\delta^*_{REg1} = \frac{\varepsilon_{g21}}{(r_g^{P_g} - 1)} = -0.0006$$
$$C_g = \frac{r_g^{P_g} - 1}{r_g^{2} - 1} = 1.55107$$
$$\left|1 - C_g\right| > 0.125$$
$$U_g(\%) = (2 * \left|1 - C_g\right| + 1) * \left|\delta^*_{REg1}\right| / S_{g1} * 100 = 1.30327$$
$$U_{gc}(\%) = \left|1 - C_g\right| * \left|\delta^*_{REg1}\right| / S_{g1} * 100 = 0.342$$

**Example 2:**

$$\varepsilon_{g21} = S_{g2} - S_{g1} = -0.0001606$$
$$\varepsilon_{g32} = S_{g3} - S_{g2} = -0.0006042$$
$$P_g = \frac{\ln(\frac{\varepsilon_{g32}}{\varepsilon_{g21}})}{\ln(r_g)} = 1.91155$$
$$\delta^*_{REg1} = \frac{\varepsilon_{g21}}{(r_g^{P_g} - 1)} = -0.00005814$$
$$C_g = \frac{r_g^{P_g} - 1}{r_g^{2} - 1} = 0.9207$$
$$\left|1 - C_g\right| < 0.125$$
$$U_g(\%) = (9.6 * (1 - C_g)^2 + 1.1) * \left|\delta^*_{REg1}\right| / S_{g1} * 100 = 0.06904$$
$$U_{gc}(\%) = (2.4 * (1 - C_g)^2 + 0.1) * \left|\delta^*_{REg1}\right| / S_{g1} * 100 = 0.006847$$