**8.14** A tornado may be modeled as the circulating flow shown in Fig. P8.14, with $\upsilon_r = \upsilon_z = 0$ and $\upsilon_\theta(r)$ such that

$$\upsilon_\theta = \begin{cases} \omega r & r \le R \\ \dfrac{\omega R^2}{r} & r > R \end{cases}$$

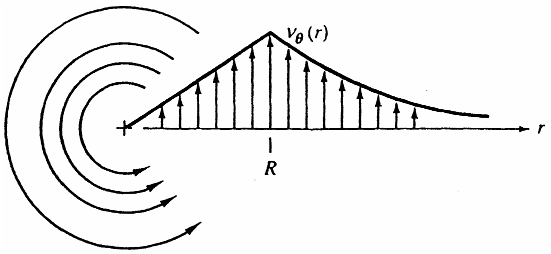


Fig. P8.14

Determine whether this flow pattern is irrotational in either the inner or outer region. Using the $r$-momentum equation (D.5) of App. D, determine the pressure distribution $p(r)$ in the tornado, assuming $p = p_\infty$ as $r \to \infty$. Find the location and magnitude of the lowest pressure.

**Solution:** The inner region is solid-body **rotation**, the outer region is **irrotational**:

$$\text{Inner region:} \quad \Omega_z = \frac{1}{r}\frac{d}{dr}(r v_\theta) = \frac{1}{r}\frac{d}{dr}(r\omega r) = \mathbf{2\boldsymbol{\omega} = constant \neq 0} \quad \textit{Ans.}\text{ (inner)}$$

$$\text{Outer region:} \quad \Omega_z = \frac{1}{r}\frac{d}{dr}(\omega R^2/r) = \mathbf{0} \quad \text{(irrotational)} \quad \textit{Ans.}\text{ (outer)}$$

The pressure is found by integrating the $r$-momentum equation (D-5) in the Appendix:

$$\frac{dp}{dr} = \rho v_\theta^2/r, \quad \text{or:} \quad p_{\text{outer}} = \int \frac{\rho}{r}\left(\frac{\omega \mathrm{R}^2}{r}\right)^2 dr = -\rho\omega^2 R^4/2r^2 + \text{constant}$$

$$\text{when} \quad r = \infty,\ p = p_\infty, \quad \text{hence} \quad \boldsymbol{p_{\text{outer}} = p_\infty - \rho\omega^2 R^4/(2r^2)} \quad \textit{Ans.}\text{ (outer)}$$

$$\text{At the match point, } r = R, \quad p_{\text{outer}} = p_{\text{inner}} = p_\infty - \rho\omega^2 R^2/2$$

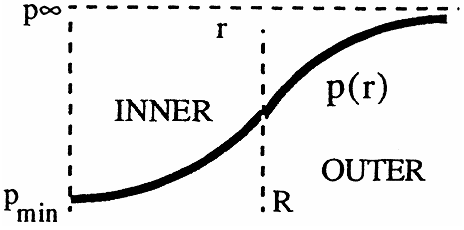
In the inner region, we integrate the radial pressure gradient and match at $r = R$:

$$p_{\text{inner}} = \int \frac{\rho}{r}(\omega r)^2 dr = \rho\omega^2 r^2/2 + \text{constant}, \quad \text{match to } p(R) = p_\infty - \rho\omega^2 R^2/2$$

$$\text{finally,} \quad \boldsymbol{p_{\text{inner}} = p_\infty - \rho\omega^2 R^2 + \rho\omega^2 r^2/2} \quad \textit{Ans.}\text{ (inner)}$$

The minimum pressure occurs at the origin, $r = 0$:

$$p_{\min} = \boldsymbol{p_\infty - \rho\omega^2 R^2} \quad \textit{Ans.}\text{ (min)}$$



**8.31** A Rankine half-body is formed as shown in Fig. P8.31. For the conditions shown, compute (a) the source strength *m* in $m^2/s$; (b) the distance *a*; (c) the distance *h*; and (d) the total velocity at point A.

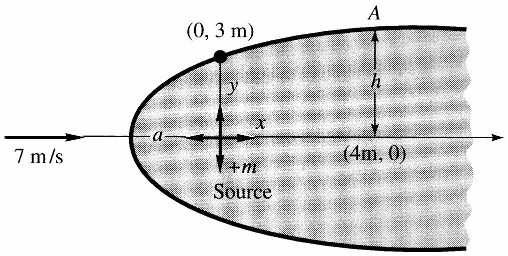


**Fig. P8.31**

**Solution:** The vertical distance above the origin is a known multiple of *m* and *a*:

$$y_{x=0} = 3\text{ m} = \frac{\pi m}{2U} = \frac{\pi m}{2(7)} = \frac{\pi a}{2},$$

$$\text{or} \quad \boldsymbol{m \approx 13.4\ \frac{m^2}{s}} \quad \text{and} \quad \boldsymbol{a \approx 1.91\text{ m}} \quad \textit{Ans.}\text{ (a, b)}$$

The distance *h* is found from the equation for the body streamline:

$$\text{At} \quad x = 4\text{ m}, \quad r_{body} = \frac{m(\pi-\theta)}{U\sin\theta} = \frac{13.4(\pi-\theta)}{7\sin\theta} = \frac{4.0}{\cos\theta}, \quad \text{solve for } \theta \approx 47.8^\circ$$

$$\text{Then} \quad r_A = 4.0/\cos(47.8^\circ) = 5.95\text{ m} \quad \text{and} \quad \mathbf{h} = r\sin\theta \approx \mathbf{4.41\ m} \quad \textit{Ans.}\text{ (c)}$$

The resultant velocity at point A is then computed from Eq. (8.18):

$$V_A = U\left(1+\frac{a^2}{r^2}+\frac{2a}{r}\cos\theta_A\right)^{1/2} = 7\left[1+\left(\frac{1.91}{5.95}\right)^2+2\left(\frac{1.91}{5.95}\right)\cos 47.8^\circ\right]^{1/2} \approx \mathbf{8.7\ \frac{m}{s}} \quad \textit{Ans.}\text{ (d)}$$

---

**8.40** Modify the Rankine oval in Fig. P8.37 so that the stream velocity and body length are the same but the thickness is unknown (not 1 m). The fluid is water at 30°C and the pressure far upstream along the body centerline is 108 kPa. Find the body thickness for which cavitation will occur at point A.

**Solution:** For water at 30°C, take $\rho$ = 996 kg/m$^3$ and $p_{vap}$ = 4242 Pa. Bernoulli's equation between far upstream and point A yields

$$\frac{p_\infty}{\rho g}+\frac{U_\infty^2}{2g}+z_\infty = \frac{p_{vap}}{\rho g}+\frac{u_{max}^2}{2g}+z_A, \quad \text{or:}$$

$$\frac{108{,}000}{996(9.81)}+\frac{10^2}{2(9.81)}+0 = \frac{4242}{996(9.81)}+\frac{[K(10)]^2}{2(9.81)}+h$$

The unknowns are the ratio K = $u_{max}/U_\infty$ and the thickness (2*h*), which depend upon the parameter ($m/U_\infty a$), which we will have to find by iteration. Trying out the answers to Prob. P8.37, $m/U_\infty a$ = 0.32, $u_{max}$ = 14.5 m/s, and *h* = 0.5 m, we find $p_A$ = 48,000 Pa, so that body is too thin, not enough pressure drop. We can iterate, or use EES, and the final solution is:

$$\frac{m}{U_\infty a} = 0.891; \quad h = 0.732m; \quad u_{\max} = 17.15\frac{m}{s}, \quad \text{Thickness} = 2h = 1.465m \quad \textit{Ans.}$$

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**8.46** A cylinder is formed by bolting two semicylindrical channels together on the inside, as shown in Fig. P8.46. There are 10 bolts per meter of width on each side, and the inside pressure is 50 kPa (gage). Using potential theory for the outside pressure, compute the tension force in each bolt if the fluid outside is sea-level air.

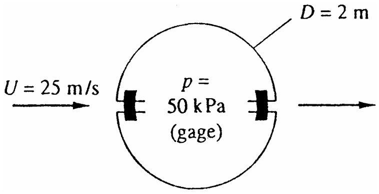

Fig. P8.46

**Solution:** For sea-level air take $\rho = 1.225$ kg/m$^3$. Use Bernoulli to find surface pressure:

$$p_\infty + \frac{\rho}{2}U_\infty^2 = 0 + \frac{1.225}{2}(25)^2 = p_s + \frac{1.225}{2}(2U_\infty \sin\theta)^2, \quad \text{or:} \quad p_s = 383 - 1531\sin^2\theta$$

$$\text{compute} \quad F_{\text{down}} = 2\int_0^{\pi/2} p\sin\theta\, b a\, d\theta = 2\int_0^{\pi/2} (383 - 1531\sin^2\theta)\sin\theta(1\,\text{m})(1\,\text{m})\,d\theta = -1276\ \frac{\text{N}}{\text{m}}$$

This is small potatoes compared to the force due to *inside* pressure:

$$F_{\text{up}} = 2p_{\text{inside}}ab = 2(50000)(1)(1) = 100000\ \frac{\text{N}}{\text{m}}$$

$$\text{Total force per meter} = 100000 - (-1276) = 101276 \div 20 \text{ bolts} \approx \mathbf{5060}\ \frac{\mathbf{N}}{\mathbf{bolt}} \quad \textit{Ans.}$$

---

**8.49** In strong winds, the force in Prob. 8.48 above can be quite large. Suppose that a hole is introduced in the hut roof at point A (see Fig. P8.48) to make $p_i$ equal to the surface pressure $p_A$. At what angle $\theta$ should hole A be placed to make the net force zero?

**Solution:** Set F = 0 in Prob. 8.48 and find the proper pressure from Bernoulli:

$$F_{\text{up}} = 0 \text{ if} \quad p_i = p_o - \frac{4}{3}\rho U_\infty^2, \quad \text{but also} \quad p_i = p_A = p_o - \frac{\rho}{2}(2U_\infty \sin\theta_A)^2$$

$$\text{Solve for} \quad \sin\theta_A = \sqrt{2/3} = 0.817 \quad \text{or} \quad \theta_A \approx \mathbf{125°} \quad \textit{Ans.}$$

(or **55°** = poor position on rear of body)

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**8.73** Set up an image system to compute the flow of a source at *unequal* distances from *two* walls, as shown in Fig. P8.73. Find the point of maximum velocity on the y-axis.

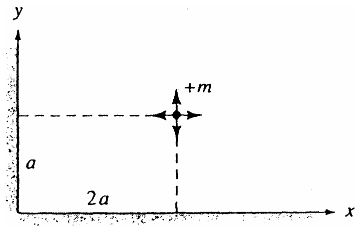
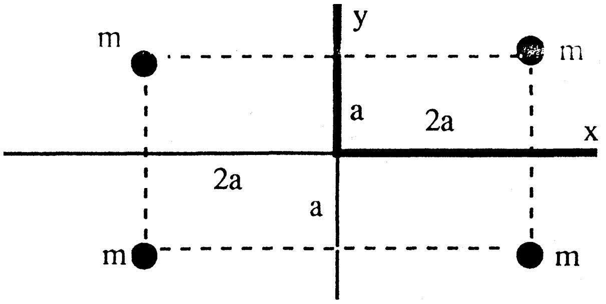


Fig. P8.73

**Solution:** Similar to Prob. 8.72 on the previous page, we place identical sources (+m) at the symmetric (but non-square) positions (x, y) = (±2a, ±a) as shown below. The induced velocity along the wall (x > 0, y = 0) has the form

$$U = \frac{2m(x+2a)}{(x+2a)^2 + a^2} + \frac{2m(x-2a)}{(x-2a)^2 + a^2}$$



This velocity has a maximum (to the *right*) at **x ≈ 2.93a**, **U ≈ 1.387 m/a**. *Ans.*

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**C8.7** Find a formula for the stream function for flow of a doublet of strength $\lambda$ at a distance *a* from a wall, as in Fig. C8.7. (a) Sketch the streamlines. (b) Are there any stagnation points? (c) Find the maximum velocity along the wall and its position.

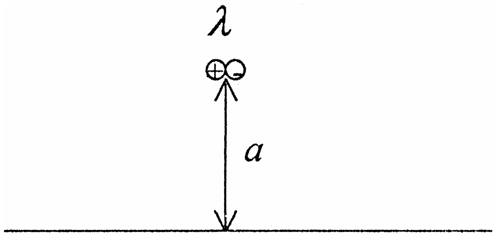
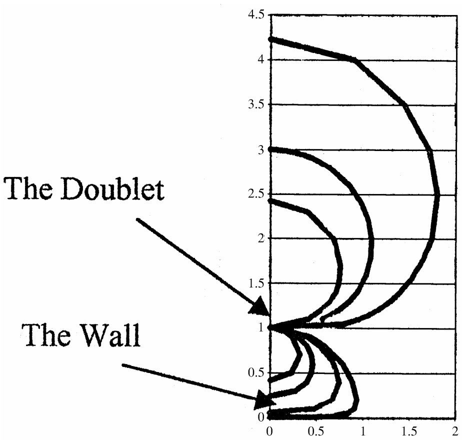


Fig. C8.7

**Solution:** Use an image doublet of the same strength and orientation at the $(x, y) = (0, -a)$. The stream function for this combined flow will form a “wall” at $y = 0$ between the two doublets:

$$\psi = -\frac{\lambda(y+a)}{x^2+(y+a)^2} - \frac{\lambda(y-a)}{x^2+(y-a)^2}$$

(a) The streamlines are shown on the next page for one quadrant of the doubly-symmetric flow field. They are fairly circular, like Fig. 8.8, above the doublet, but they flatten near the wall.



**Problem C8.7**

(b) There are **no stagnation points** in this flow field. *Ans.* (b)

(c) The velocity along the wall ($y = 0$) is found by differentiating the stream function:

$$u_{wall} = \frac{\partial\psi}{\partial y}\Big|_{y=0} = -\frac{\lambda}{x^2+a^2} + \frac{2\lambda a^2}{(x^2+a^2)^2} - \frac{\lambda}{x^2+a^2} + \frac{2\lambda a^2}{(x^2+a^2)^2}$$

The maximum velocity occurs at $x = 0$, that is, right between the two doublets:

$$u_{w,\max} = \frac{2\lambda}{a^2} \quad \textit{Ans. } \text{(c)}$$