P7.6 For the laminar parabolic boundary-layer profile of Eq. (7.6), compute the shape factor “H” and compare with the exact Blasius-theory result, Eq. (7.31).

Solution: Given the profile approximation $u/U \approx 2\eta - \eta^2$, where $\eta = y/\delta$, compute

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \delta\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta = \frac{2}{15}\delta$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta\int_0^1 (1-2\eta+\eta^2)\,d\eta = \frac{1}{3}\delta$$

Hence $H = \delta^*/\theta = (\delta/3)/(2\delta/15) \approx$ **2.5** (compared to *2.59* for Blasius solution)

P7.10 Repeat Prob. 7.9, using the polynomial profile suggested by K. Pohlhausen in 1921:

$$\frac{u}{U} \approx 2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}$$

Does this profile satisfy the boundary conditions of laminar flat-plate flow?

Solution: Pohlhausen’s quadratic profile satisfies no-slip at the wall, a smooth merge with $u \to U$ as $y \to \delta$, and, further, the boundary-layer curvature condition at the wall. From Eq. (7.19b),

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}\right)_{wall} = 0, \quad \text{or:} \quad \frac{\partial^2 u}{\partial y^2}\Big|_{wall} = 0 \quad \text{for flat-plate flow} \left(\frac{\partial p}{\partial x} = 0\right)$$

This profile gives the following integral approximations:

$$\theta \approx \frac{37}{315}\delta; \quad \delta^* \approx \frac{3}{10}\delta; \quad \tau_w \approx \mu\frac{2U}{\delta} \approx \rho U^2 \frac{d}{dx}\left(\frac{37}{315}\delta\right), \quad \text{integrate to obtain:}$$

$$\frac{\delta}{x} \approx \frac{\sqrt{(1260/37)}}{\sqrt{Re_x}} \approx \frac{\mathbf{5.83}}{\sqrt{\mathbf{Re_x}}}; \quad C_f = \frac{\theta}{x} \approx \frac{\mathbf{0.685}}{\sqrt{\mathbf{Re_x}}};$$

$$\frac{\delta^*}{x} \approx \frac{\mathbf{1.751}}{\sqrt{\mathbf{Re_x}}}; \quad H \approx \mathbf{2.554} \quad \textit{Ans.}\ (a, b, c, d)$$

P7.24 Air at 20°C and 1 atm flows past the flat plate in Fig. P7.24. The two pitot tubes are each 2 mm from the wall. The manometer fluid is water at 20°C. If U = 15 m/s and L = 50 cm, determine the values of the manometer readings h_1 and h_2 in cm. Assume laminar boundary-layer flow.

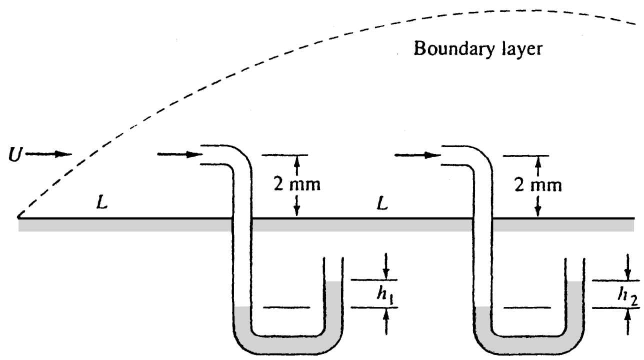


Fig. P7.24

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. The velocities *u* at each pitot inlet can be estimated from the Blasius solution:

$$(1)\quad \eta_1 = y[U/\nu x_1]^{1/2} = (0.002)\{15/[1.5E{-}5(0.5)]\}^{1/2} = 2.83,\quad \text{Table 7.1: read } f' \approx 0.816$$

$$\text{Then}\quad u_1 = Uf' = 15(0.816) \approx 12.25 \text{ m/s}$$

$$(2)\quad \eta_2 = y[U/\nu x_2]^{1/2} = 2.0,\quad f' \approx 0.630,\quad u_2 = 15(0.630) \approx 9.45 \text{ m/s}$$

Assume constant stream pressure, then the manometers are a measure of the local velocity *u* at each position of the pitot inlet, so we can find Δp across each manometer:

$$\Delta p_1 = \frac{\rho}{2}u_1^2 = \frac{1.2}{2}(12.25)^2 = 90 \text{ Pa} = \Delta\rho g h_1 = (998 - 1.2)(9.81)h_1,\quad \mathbf{h_1 \approx 9.2\ mm}$$

$$\Delta p_2 = \frac{\rho}{2}u_2^2 = \frac{1.2}{2}(9.45)^2 = 54 \text{ Pa} = (998 - 1.2)(9.81)h_2,\quad \text{or:}\quad \mathbf{h_2 \approx 5.5\ mm}\quad \textit{Ans.}$$

P7.29 Let the flow straighteners in Fig. P7.28 form an array of 20 × 20 boxes of size *a* = 4 cm and *L* = 25 cm. If the approach velocity is U_o = 12 m/s and the fluid is sea-level standard air, estimate (a) the total array drag and (b) the pressure drop across the array. Compare with Sec. 6.6.

Solution: For sea-level air, take ρ = 1.205 kg/m^3 and μ = 1.78E–5 kg/m·s. The analytical formulas for array drag and pressure drop are given above. Hence

$$F_{array} = 2.656N^2(\rho\mu L)^{1/2}U^{3/2}a = 2.656(20)^2[1.205(1.78E{-}5)(0.25)]^{1/2}(12)^{3/2}(0.04)$$

$$\text{or:}\quad \mathbf{F \approx 4.09\ N}\quad (Re_L = 203000, \text{OK, laminar})\quad \textit{Ans. (a)}$$

$$\Delta p_{array} = \frac{F}{(Na)^2} = \frac{4.09}{[20(0.04)]^2} \approx \mathbf{6.4\ Pa}\quad \textit{Ans. (b)}$$

This is a far cry from the (much lower) estimate would have by assuming the array is a bunch of long square ducts as in Sect. 6.6 (as shown in Prob. 7.28):

$$\Delta p_{\text{long duct}} \approx \frac{28.5\mu LU}{a^2} = \frac{28.5(1.78E{-}5)(0.25)(12)}{(0.04)^2} \approx \mathbf{0.95\ Pa} \quad \text{(not accurate)} \quad \textit{Ans.}$$

P7.32 A flat plate of length *L* and height δ is placed at a wall and is parallel to an approaching boundary layer, as in Fig. P7.32. Assume that the flow over the plate is fully turbulent and that the approaching flow is a one-seventh-power law

$$u(y) = U_o\left(\frac{y}{\delta}\right)^{1/7}$$

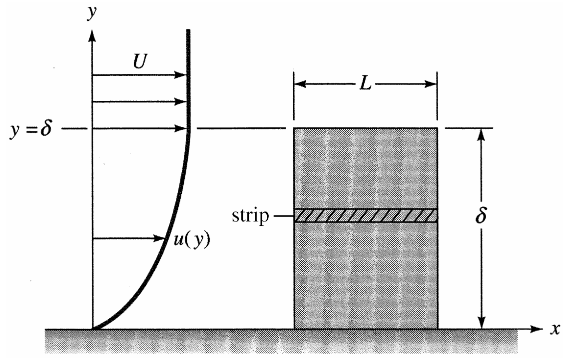


Fig. P7.32

Using strip theory, derive a formula for the drag coefficient of this plate. Compare this result with the drag of the same plate immersed in a uniform stream *U*o.

Solution: For a 'strip' of plate *dy* high and *L* long, subjected to flow *u*(y), the force is

$$dF = C_D\frac{\rho}{2}u^2(L\,dy)(2\text{ sides}), \quad \text{where } C_D \approx \frac{0.031}{(\rho uL/\mu)^{1/7}}, \quad \text{combine into dF and integrate:}$$

$$dF = 0.031\rho\nu^{1/7}L^{6/7}u^{13/7}dy, \quad \text{or} \quad F = 0.031\rho\nu^{1/7}L^{6/7}\int_0^{\delta}\left[U_o(y/\delta)^{1/7}\right]^{13/7}dy$$

The result is $\mathbf{F = 0.031(49/62)\rho\nu^{1/7}L^{6/7}U_O^{13/7}\delta}$ *Ans.*

This drag is (49/62), or 79%, of the force on the same plate immersed in a uniform stream.

P7.43 In the flow of air at 20°C and 1 atm past a flat plate in Fig. P7.43, the wall shear is to be determined at position *x* by a *floating element* (a small area connected to a strain-gage force measurement). At $x = 2$ m, the element indicates a shear stress of 2.1 Pa. Assuming turbulent flow from the leading edge, estimate (a) the stream velocity *U*, (b) the boundary layer thickness δ at the element, and (c) the boundary-layer velocity *u*, in m/s, at 5 cm above the element.

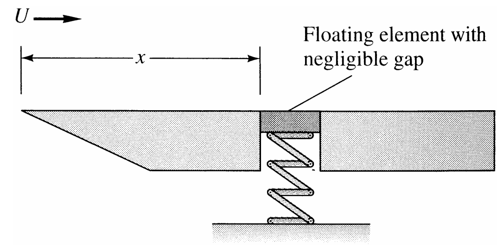


Fig. P7.43

Solution: For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E–}5$ kg/m·s. The shear stress is

$$\tau_w = 2.1\text{ Pa} = C_f \frac{\rho}{2}U^2 = \frac{0.027}{(\rho U x/\mu)^{1/7}}\left(\frac{\rho U^2}{2}\right) = \frac{0.027}{[1.2U(2)/1.8E\text{–}5]^{1/7}}\left(\frac{1.2U^2}{2}\right)$$

$$\text{Solve for}\quad U \approx \mathbf{34\ \frac{m}{s}}\quad \textit{Ans. (a)}\quad \text{Check}\ \ Re_x \approx 4.54E6\quad (\text{OK, turbulent})$$

With the local Reynolds number known, solve for local thickness:

$$\delta \approx \frac{0.16x}{Re_x^{1/7}} = \frac{0.16(2\text{ m})}{(4.54E6)^{1/7}} \approx 0.036\text{ m} \approx \mathbf{36\ mm}\quad \textit{Ans. (b)}$$

Normally, the log-law, Eq. (7.34), is probably best for estimating the velocity at y = 5 cm above the element. However, from Ans. (*b*) just above, we see that this point is <u>outside</u> the boundary layer. Therefore, the velocity must be **u = U ≈ 34 m/s**. *Ans.* (c).
[NOTE: Part (c) was supposed to state y = 5 **<u>mm</u>**, in which case the correct answer would have been u ≈ *26.5* m/s.]

P7.50 Consider the flat-walled diffuser in Fig. P7.50, which is similar to that of Fig. 6.26*a* with constant width *b*. If *x* is measured from the inlet and the wall boundary layers are thin, show that the core velocity *U*(*x*) in the diffuser is given approximately by

$$U = \frac{U_o}{1 + (2x\tan\theta)/W}$$

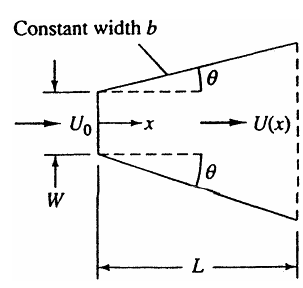


Fig. P7.50

where *W* is the inlet height. Use this velocity distribution with Thwaites' method to compute the wall angle θ for which laminar separation will occur in the exit plane when diffuser length $L = 2W$. Note that the result is independent of the Reynolds number.

Solution: We can approximate U(x) by the one-dimensional continuity relation:

$$U_o W b = U(W + 2x\tan\theta)b,\ \text{or:}\ U(x) \approx U_o/[1 + 2x\tan\theta/W]\quad (\text{same as Görtler, Prob. 7.48})$$

We return to the solution from Görtler's (n = 1) distribution in Prob. 7.48:

$$\lambda = -0.09 \quad \text{if} \quad \frac{2x\tan\theta}{W} = 0.159 \ \ \text{(separation)}, \quad \text{or} \quad x = L = 2W,$$

$$\tan\theta_{sep} = \frac{0.159}{4} = 0.03975, \quad \boldsymbol{\theta_{sep}} \approx 2.3° \quad \textit{Ans.}$$

[This laminar result is much less than the <u>turbulent</u> value $\theta_{sep} \approx 8° - 10°$ in Fig. 6.26c.]

P7.85 An aluminum cylinder (SG = 2.7) slides concentrically down a taut 1-mm-diameter wire as shown in the figure. Its length is L = 8 cm and its radius R = 1 cm. A 2-mm-diameter hole down the cylinder center is lubricated by SAE 30 oil at 20°C. Estimate the terminal fall velocity V if ambient air drag is (a) neglected; or (b) included. Assume air at 1 atm and 20°C.

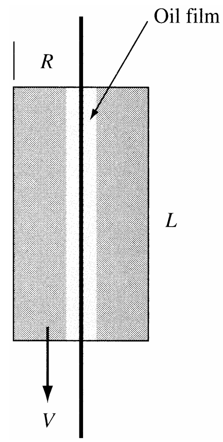


Fig. P7.85

Solution: For SAE 30 oil, from Table A-3, $\mu_{oil} \approx 0.29$ kg/m·s. Calculate the weight of the cylinder:

$$W = \rho_{alum} g\pi\left(R^2 - r_{hole}^2\right)L = [2.7(998)](9.81)\pi(0.01^2 - 0.001^2)(0.08) = 0.658 \text{ N}$$

From Problem 4.89, the (laminar) shear stress at the inner wall of the cylinder is

$$\tau_{w\text{-}inner} = \frac{\mu V}{r_{hole}\ln\left(\frac{r_{hole}}{r_{wire}}\right)} = \frac{0.29V}{0.001\ln(2)} \approx 418V \quad \left(with\, V\, in\ \frac{\text{m}}{\text{s}}\right)$$

(a) If air drag is neglected, the oil-stress force balances the cylinder weight:

$$W = 0.658\ N = \tau_w 2\pi r_{hole} L = (418V)2\pi(0.001)(0.08),$$

$$\text{Solve for} \quad V_{oil\text{-}only} \approx \mathbf{3.13\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

(b) For air take $\rho_{air} = 1.2$ kg/m^3. From Table 7-3 for flat cylinder, $C_D \approx 0.99$. Thus

$$W = 0.658 = \tau_w 2\pi r_{hole} L + C_D \frac{\rho_{air}}{2} V^2 \pi R^2 = 0.210V + 0.000187V^2$$

$$\text{Rearrange:} \quad V^2 + 1127V - 3525 = 0, \quad \text{solve} \quad V_{oil+air} \approx \mathbf{3.12\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(b)}$$

We see that air drag is negligible in this thick-oil, low-speed situation.

P7.91 A cup anemometer uses two 5-cm-diameter hollow hemispheres connected to two 15-cm rods, as in Fig. P7.91. Rod drag is neglected, and the central bearing has a retarding torque of 0.004 N·m. With simplifying assumptions, estimate and plot rotation rate Ω versus wind velocity in the range 0 < U < 25 m/s.

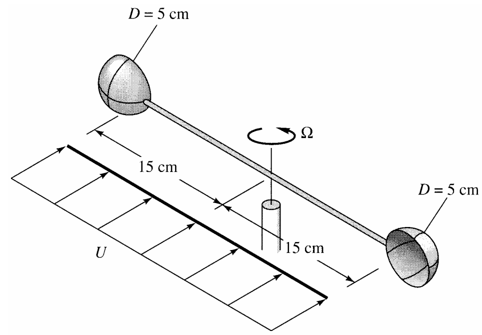
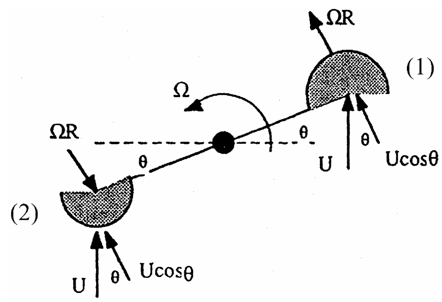


Fig. P7.91

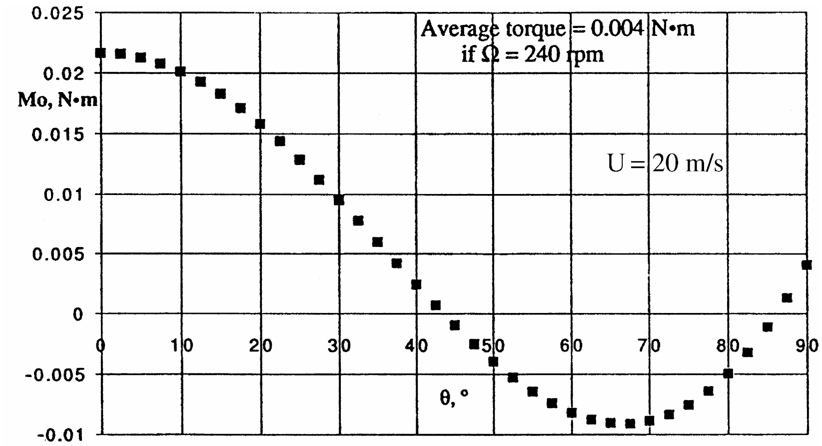
Solution: For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E–5 kg/m·s. For any instantaneous angle θ, as shown, the drag forces are assumed to depend on the relative velocity normal to the cup:

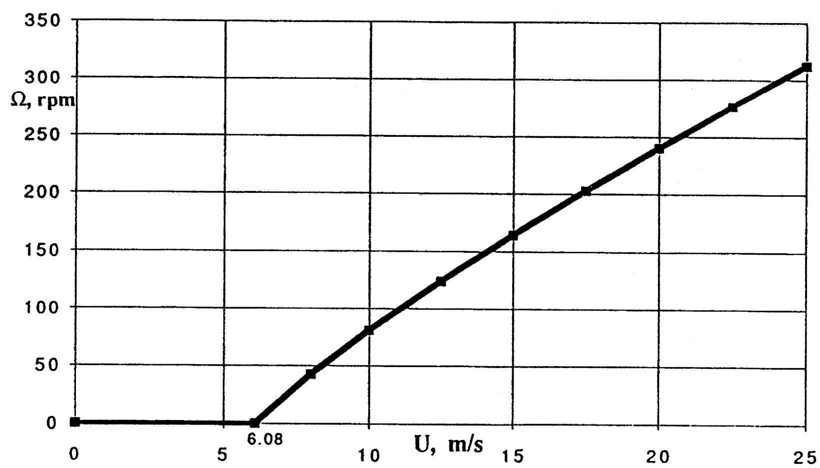


$$M_o = R\left[C_{D1}\frac{\rho}{2}(U\cos\theta - \Omega R)^2 - C_{D2}\frac{\rho}{2}(U\cos\theta + \Omega R)^2\right],$$

$$C_{D1} \approx 1.4, \quad C_{D2} \approx 0.4$$

For a given wind velocity 0 < U < 25 m/s, we find the rotation rate Ω (here in rad/s) for which the average torque over a 90° sweep is exactly equal to the frictional torque of 0.004 N·m. [The torque given by the formula mirrors itself over 90° increments.] For U = 20 m/s, the torque variation given by the formula is shown in the graph below. We do this for the whole range of U values and then plot Ω (in rev/min) versus U below. We see that the anemometer will not rotate until U ≥ 6.08 m/s. Thereafter the variation of Ω with U is approximately linear, making this a popular wind-velocity instrument.





P7.99 Two steel balls (SG = 7.86) are connected by a thin hinged rod of negligible weight and drag, as shown in Fig. P7.99. A stop prevents counter-clockwise rotation. Estimate the sea-level air velocity U for which the rod will first begin to rotate clockwise.

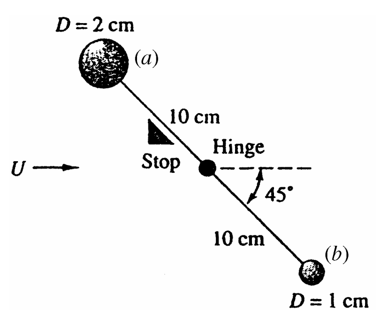


Fig. P7.99

Solution: For sea-level air, take $\rho = 1.225$ kg/m^3 and μ = 1.78E–5 kg/m·s. Let "a" and "b" denote the large and small balls, respectively, as shown. The rod begins to rotate when the moments of drag and weight are balanced. The (clockwise) moment equation is

$$\Sigma M_o = F_a(0.1\sin 45°) - W_a(0.1\cos 45°) - F_b(0.1\sin 45°) + W_b(0.1\cos 45°) = 0$$

For 45°, there are nice cancellations to obtain $\therefore\ F_a - F_b = W_a - W_b$, or:

$$C_{Da}\frac{\rho}{2}U^2\frac{\pi}{4}D_a^2 - C_{Db}\frac{\rho}{2}U^2\frac{\pi}{4}D_b^2 \stackrel{?}{=} (SG)\rho_{water}g\frac{\pi}{6}D_a^3 - (SG)\rho_{water}g\frac{\pi}{6}D_b^3$$

Assuming that $C_{Da} = C_{Db} \approx 0.47$ (Re < 250000), we may easily solve for air velocity:

$$U^2(0.47)\left(\frac{1.225}{2}\right)\frac{\pi}{4}[(0.02)^2 - (0.01)^2] = (7.86)(9790)\frac{\pi}{6}[(0.02)^3 - (0.01)^3]$$

$$\text{Solve for} \quad U = \sqrt{4158} \approx \mathbf{64\ m/s} \quad \textit{Ans.}$$

We may check that $Re_{max} = 1.225(64)(0.02)/1.78E{-5} \approx$ *89000*, OK, $C_D \approx 0.47$.

P7.110 A baseball pitcher throws a curveball with an initial velocity of 65 mi/h and a spin of 6500 r/min about a vertical axis. A baseball weighs 0.32 lbf and has a diameter of 2.9 in. Using the data of Fig. P7.108 for turbulent flow, estimate how far such a curveball will have deviated from its straightline path when it reaches home plate 60.5 ft away.

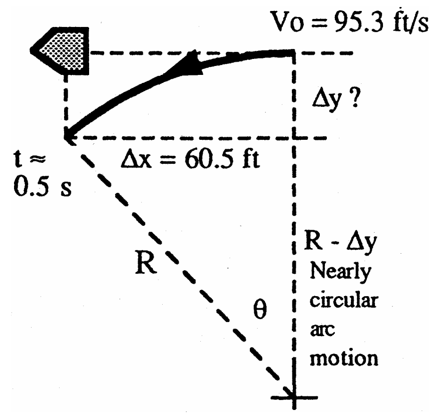


Fig. P7.110

Solution: For sea-level air, take ρ = 0.00238 slug/ft^3 and μ = 3.72E−7 in nearly a circular arc, as shown above. However, gravity is *not* involved in this curved *horizontal* path. First evaluate the lift and drag:

$$V_o = 65\ \frac{mi}{h} = 95\ \frac{ft}{s}, \quad \omega = 6500\left(\frac{2\pi}{60}\right) = 681\ \frac{rad}{s}, \quad \frac{\omega R}{V} = \frac{681(2.9/24)}{95} \approx 0.86$$

$$\text{Fig. P7.108:} \quad \text{Read } C_D \approx \mathbf{0.60},\ C_L \approx \mathbf{0.17}$$

The initial accelerations in the *x*- and *y*-directions are

$$a_{x,0} = -\frac{drag}{m} = -\frac{0.44(0.00238/2)(95)^2(\pi/4)(2.9/12)^2}{0.32/32.2\ slug} \approx \mathbf{-29.7\ \frac{ft}{s^2}}$$

$$a_{y,0} = -\frac{lift}{m} = -\frac{0.17(0.00238/2)(95)^2(\pi/4)(2.9/12)^2}{0.32/32.2} \approx \mathbf{-8.5\ \frac{ft}{s^2}}$$

The ball is in flight about 0.5 sec, so a_x causes it to slow down to about 85 ft/s, with an average velocity of (95 + 85)/2 ≈ **90** ft/s. Then one can use these numbers to estimate R:

$$R = \frac{V^2_{avg}}{|a_y|} = \frac{(90)^2}{8.5} \approx 954\ ft; \quad \theta = \sin^{-1}\left(\frac{\Delta x}{R}\right) = \sin^{-1}\left(\frac{60.5}{954}\right) \approx 3.63°$$

$$\text{Finally,} \quad \Delta y_{home\ plate} = R(1-\cos\theta) = 954(1-\cos 3.63°) \approx \mathbf{1.9\ ft} \quad \textit{Ans.}$$

C7.4 Consider a simple pendulum with an unusual bob shape: a cup of diameter D whose axis is in the plane of oscillation. Neglect the mass and drag of the rod L. (a) Set up the differential equation for θ(t) and (b) non-dimensionalize this equation. (c) Determine the natural frequency for $\theta \ll 1$. (d) For L = 1 m, D = 1 cm, m = 50 g, and air at 20°C and 1 atm, and θ(0) = 30°, find (numerically) the time required for the oscillation amplitude to drop to 1°.

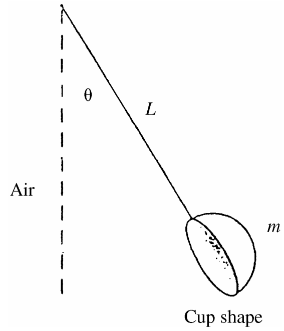


Fig. C7.4

Solution: (a) Let $L_{eq} = L + D/2$ be the effective length of the pendulum. Sum forces in the direction of the motion of the bob and rearrange into the basic 2nd-order equation:

$$\sum F_{tangential} = -mg\sin\theta - C_D\frac{\rho}{2}V_t^2\frac{\pi}{4}D^2 = \mathrm{m}\frac{dV_t}{dt}, \quad \text{where } V_t = L_{eq}\frac{d\theta}{dt}$$

$$\text{Rearrange:}\quad \boldsymbol{\ddot{\theta} + K\dot{\theta}^2 + \frac{g}{L_{eq}}\sin\theta = 0}, \quad \text{where } K = \frac{C_D\rho L_{eq}\pi D^2}{8\text{ m}} \quad \textit{Ans.}\text{ (a)}$$

Note that $C_D \approx$ **0.4** when moving to the right and about **1.4** moving to the left (Table 7.3).

(b) Now θ is already dimensionless, so define dimensionless time $\tau = t(g/L_{eq})^{1/2}$ and substitute into the differential equation above. We obtain the dimensionless result

$$\boldsymbol{\frac{d^2\theta}{d\tau^2} + K\left(\frac{d\theta}{d\tau}\right)^2 + \theta = 0} \quad \textit{Ans.}\text{ (b)}$$

Thus the only dimensionless parameter is ***K*** from part (a) above.
(c) For $\theta \ll 1$, the term involving K is neglected, and $\sin\theta \approx \theta$ itself. We obtain

$$\ddot{\theta} + \omega_n^2\theta \approx 0, \quad \text{where } \boldsymbol{\omega_n = \sqrt{\frac{g}{L_{eq}}}} \quad \textit{Ans.}\text{ (c)}$$

Thus the natural frequency is $(g/L_{eq})^{1/2}$ just as for the simple drag-free pendulum. Recall that $L_{eq} = L + D/2$. Note again that K has a different value when moving to the right ($C_D \approx 0.4$) or to the left ($C_D \approx 1.4$).

(d) For the given data, $\rho_{air} = 1.2$ kg/m^3, $L_{eq} = L + D/2 = 1.05$ m, and the parameter K is

$$K = \frac{C_D(1.2)(1.05)\pi(0.1)^2}{8(0.050)} = 0.099C_D = 0.0396 \ \ (\textit{moving to the right})$$

$= 0.1385$ (*moving to the left*)

The differential equation from part (b) is then solved for $\theta(0) = 30° = \pi/6$ radians. The natural frequency is $(9.81/1.05)^{1/2} = 3.06$ rad/s, with a dimensionless period of 2π. Integrate numerically, with Runge-Kutta or MatLab or Excel or whatever, until $\theta = 1° = \pi/180$ radians. The time-series results are shown in the figure below.

We see that the pendulum is very *lightly damped*—drag forces are only about 1/50th of the weight of the bob. After ten cycles, the amplitude has only dropped to 22.7°—we will never get down to 1° in the lifetime of my computer. The dimensionless period is 6.36, or only 1% greater than the simple drag-free theoretical value of 2π.