

6.4 For flow of SAE 30 oil through a 5-cm-diameter pipe, from Fig. A.1, for what flow rate in m^3/h would we expect transition to turbulence at (a) 20°C and (b) 100°C ?

Solution: For SAE 30 oil take $\rho = 891 \text{ kg/m}^3$ and take $\mu = 0.29 \text{ kg/m}\cdot\text{s}$ at 20°C (Table A.3) and $0.01 \text{ kg/m}\cdot\text{s}$ at 100°C (Fig A.1). Write the critical Reynolds number in terms of flow rate Q :

$$(a) \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891 \text{ kg/m}^3)Q}{\pi(0.29 \text{ kg/m}\cdot\text{s})(0.05 \text{ m})},$$

$$\text{solve } Q = 0.0293 \frac{\text{m}^3}{\text{s}} = \mathbf{106 \frac{\text{m}^3}{\text{h}}} \text{ Ans. (a)}$$

$$(b) \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891 \text{ kg/m}^3)Q}{\pi(0.010 \text{ kg/m}\cdot\text{s})(0.05 \text{ m})},$$

$$\text{solve } Q = 0.00101 \frac{\text{m}^3}{\text{s}} = \mathbf{3.6 \frac{\text{m}^3}{\text{h}}} \text{ Ans. (b)}$$

6.21 In Tinyland, houses are less than a foot high! The rainfall is laminar! The drainpipe in Fig. P6.21 is only 2 mm in diameter. (a) When the gutter is full, what is the rate of draining? (b) The gutter is designed for a sudden rainstorm of up to 5 mm per hour. For this condition, what is the maximum roof area that can be drained successfully? (c) What is Re_d ?

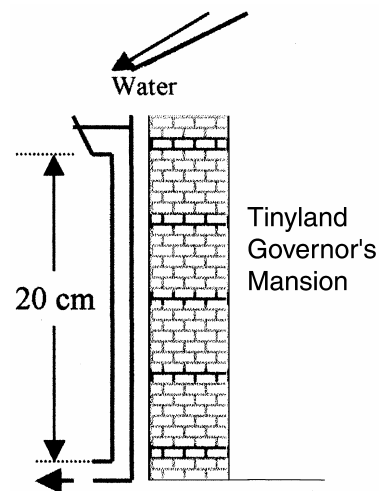


Fig. P6.21

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

$$\Delta z = \frac{V^2}{2g} + h_f, \quad \text{where } h_{f,\text{laminar}} = \frac{32\mu LV}{\rho g d^2}$$

For water, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. (a) With Δz known, this is a quadratic equation for the pipe velocity V :

$$0.2 \text{ m} = \frac{V^2}{2(9.81 \text{ m/s}^2)} + \frac{32(0.001 \text{ kg/m}\cdot\text{s})(0.2 \text{ m})V}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.002 \text{ m})^2},$$

$$\text{or: } 0.051V^2 + 0.1634V - 0.2 = 0, \quad \text{Solve for } V = 0.945 \frac{\text{m}}{\text{s}},$$

$$Q = \frac{\pi}{4}(0.002 \text{ m})^2 \left(0.945 \frac{\text{m}}{\text{s}}\right) = 2.97\text{E-}6 \frac{\text{m}^3}{\text{s}} = \mathbf{0.0107 \frac{\text{m}^3}{\text{h}}} \text{ Ans. (a)}$$

(b) The roof area needed for maximum rainfall is $0.0107 \text{ m}^3/\text{h} \div 0.005 \text{ m/h} = \mathbf{2.14 \text{ m}^2}$. Ans. (b)

(c) The Reynolds number of the gutter is $\text{Re}_d = (998)(0.945)(0.002)/(0.001) = \mathbf{1890}$ laminar. Ans. (c)

6.29 Oil, with $\rho = 890 \text{ kg/m}^3$ and $\mu = 0.07 \text{ kg/m}\cdot\text{s}$, flows through a horizontal pipe 15 m long. The power delivered to the flow is 1 hp. (a) What is the appropriate pipe diameter if the flow is at the laminar transition point? For this condition, what are (b) Q in m^3/h ; and (c) τ_w in kPa?

Solution: (a, b) Set the Reynolds number equal to 2300 and the (laminar) power equal to 1 hp:

$$\text{Re}_d = 2300 = \frac{(890 \text{ kg/m}^3)Vd}{0.07 \text{ kg/m}\cdot\text{s}} \quad \text{or} \quad Vd = 0.181 \text{ m}^2/\text{s}$$

$$\text{Power} = 1 \text{ hp} = 745.7 \text{ W} = Q\Delta p_{\text{laminar}} = \left(\frac{\pi}{4}d^2V\right)\left(\frac{32\mu LV}{d^2}\right) = \left(\frac{\pi}{4}\right)32(0.07)(15)V^2$$

$$\text{Solve for } V = 5.32 \frac{\text{m}}{\text{s}} \quad \text{and} \quad \mathbf{d = 0.034 \text{ m}} \quad \text{Ans. (a)}$$

It follows that $Q = (\pi/4)d^2V = (\pi/4)(0.034 \text{ m})^2(5.32 \text{ m/s}) = 0.00484 \text{ m}^3/\text{s} = \mathbf{17.4 \text{ m}^3/\text{h}}$ Ans. (b)

(c) From Eq. (6.12), the wall shear stress is

$$\tau_w = \frac{8\mu V}{d} = \frac{8(0.07 \text{ kg/m}\cdot\text{s})(5.32 \text{ m/s})}{(0.034 \text{ m})} = 88 \text{ Pa} = \mathbf{0.088 \text{ kPa}} \quad \text{Ans. (c)}$$

6.37 Two infinite plates a distance h apart are parallel to the xz plane with the upper plate moving at speed V , as in Fig. P6.37. There is a fluid of viscosity μ and constant pressure between the plates. Neglecting gravity and assuming incompressible turbulent flow $u(y)$ between the plates, use the logarithmic law and appropriate boundary conditions to derive a formula for dimensionless wall shear stress versus dimensionless plate velocity. Sketch a typical shape of the profile $u(y)$.

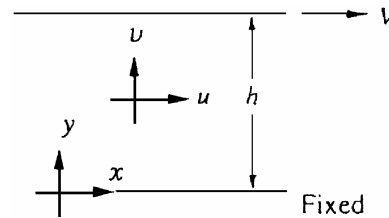
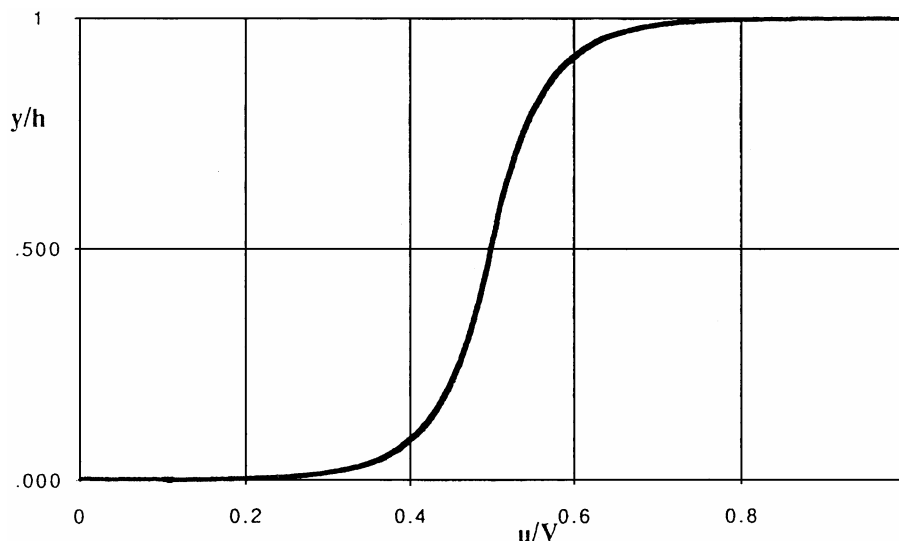


Fig. P6.37

Solution: The shear stress between parallel plates is *constant*, so the centerline velocity must be exactly $u = V/2$ at $y = h/2$. Anti-symmetric log-laws form, one with increasing velocity for $0 < y < h/2$, and one with decreasing velocity for $h/2 < y < h$, as shown below:



The match-point at the center gives us a log-law estimate of the shear stress:

$$\frac{V}{2u^*} \approx \frac{1}{\kappa} \ln\left(\frac{hu^*}{2\nu}\right) + B, \quad \kappa \approx 0.41, B \approx 5.0, u^* = (\tau_w/\rho)^{1/2} \quad \text{Ans.}$$

This is one form of “dimensionless shear stress.” The more normal form is friction coefficient versus Reynolds number. Calculations from the log-law fit a Power-law curve-fit expression in the range $2000 < \text{Re}_h < 1\text{E}5$:

$$C_f = \frac{\tau_w}{(1/2)\rho V^2} \approx \frac{0.018}{(\rho V h/\nu)^{1/4}} = \frac{\mathbf{0.018}}{\mathbf{\text{Re}_h^{1/4}}} \quad \text{Ans.}$$

6.40 Theodore von Kármán in 1930 theorized that turbulent shear could be represented by $\tau_{\text{turb}} = \varepsilon \, du/dy$ where $\varepsilon = \rho \kappa^2 y^2 |du/dy|$ is called the *mixing-length eddy viscosity* and $\kappa \approx 0.41$ is Kármán’s dimensionless *mixing-length constant* [2,3]. Assuming that $\tau_{\text{turb}} \approx \tau_w$ near the wall, show that this expression can be integrated to yield the logarithmic-overlap law, Eq. (6.28).

Solution: This is accomplished by straight substitution:

$$\tau_{\text{turb}} \approx \tau_w = \rho u^{*2} = \varepsilon \frac{du}{dy} = \left[\rho \kappa^2 y^2 \left| \frac{du}{dy} \right| \right] \frac{du}{dy}, \quad \text{solve for } \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Integrate: } \int du = \frac{u^*}{\kappa} \int \frac{dy}{y}, \quad \text{or: } \mathbf{u = \frac{u^*}{\kappa} \ln(y) + \text{constant}} \quad \text{Ans.}$$

To convert this to the exact form of Eq. (6.28) requires fitting to experimental data.

6.54* A swimming pool W by Y by h deep is to be emptied by gravity through the long pipe shown in Fig. P6.54. Assuming an average pipe friction factor f_{av} and neglecting minor losses, derive a formula for the time to empty the tank from an initial level h_0 .

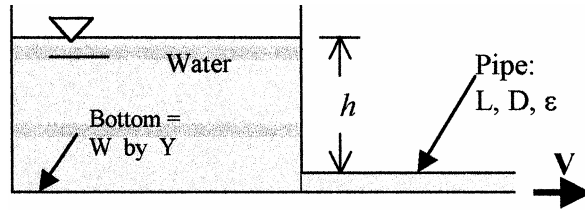


Fig. P6.54

Solution: With no driving pressure and negligible tank surface velocity, the energy equation can be combined with a control-volume mass conservation:

$$h(t) = \frac{V^2}{2g} + f_{av} \frac{L}{D} \frac{V^2}{2g}, \quad \text{or:} \quad Q_{out} = A_{pipe} V = \frac{\pi}{4} D^2 \sqrt{\frac{2gh}{1 + f_{av}L/D}} = -WY \frac{dh}{dt}$$

We can separate the variables and integrate for time to drain:

$$\frac{\pi}{4} D^2 \sqrt{\frac{2g}{1 + f_{av}L/D}} \int_0^t dt = -WY \int_{h_o}^0 \frac{dh}{\sqrt{h}} = -WY (0 - 2\sqrt{h_o})$$

Clean this up to obtain: $t_{drain} \approx \frac{4WY}{\pi D^2} \sqrt{\frac{2h_o(1 + f_{av}L/D)}{g}}$ Ans.

6.76 The small turbine in Fig. P6.76 extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m³/h. Sketch the EGL and HGL accurately.

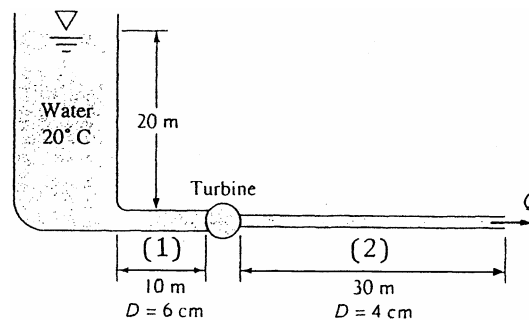


Fig. P6.76

Solution: For water, take $\rho = 998$ kg/m³ and $\mu = 0.001$ kg/m·s. For wrought iron, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d_1 = 0.046/60 \approx 0.000767$ and $\varepsilon/d_2 = 0.046/40 \approx 0.00115$. The energy equation, with $V_1 \approx 0$ and $p_1 = p_2$, gives

$$z_1 - z_2 = 20 \text{ m} = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{turbine}, \quad h_{f1} = f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g} \quad \text{and} \quad h_{f2} = f_2 \frac{L_2}{d_2} \frac{V_2^2}{2g}$$

$$\text{Also, } h_{turbine} = \frac{P}{\rho g Q} = \frac{400 \text{ W}}{998(9.81)Q} \quad \text{and} \quad Q = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

The only unknown is Q , which we may determine by iteration after an initial guess:

$$h_{\text{turb}} = \frac{400}{998(9.81)Q} = 20 - \frac{8f_1L_1Q^2}{\pi^2gd_1^5} - \frac{8f_2L_2Q^2}{\pi^2gd_2^5} - \frac{8Q^2}{\pi^2gd_2^4}$$

Guess $Q = 0.003 \frac{\text{m}^3}{\text{s}}$, then $Re_1 = \frac{4\rho Q}{\pi\mu d_1} = 63500$, $f_{1,\text{Moody}} \approx 0.0226$,

$$Re_2 = 95300, \quad f_2 \approx 0.0228.$$

But, for this guess, $h_{\text{turb}}(\text{left hand side}) \approx 13.62 \text{ m}$, $h_{\text{turb}}(\text{right hand side}) \approx 14.53 \text{ m}$ (wrong). Other guesses converge to $h_{\text{turb}} \approx 9.9 \text{ meters}$. For $Q \approx 0.00413 \text{ m}^3/\text{s} \approx 15 \text{ m}^3/\text{h}$. *Ans.*

6.83 For the system of Fig. P6.55, let $\Delta z = 80 \text{ m}$ and $L = 185 \text{ m}$ of cast-iron pipe. What is the pipe diameter for which the flow rate will be $7 \text{ m}^3/\text{h}$?

Solution: For water, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. For cast iron, take $\varepsilon \approx 0.26 \text{ mm}$, but d is unknown. The energy equation is simply

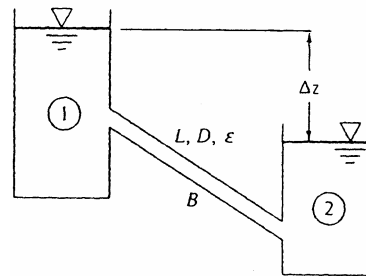


Fig. P6.55

$$\Delta z = 80 \text{ m} = h_f = \frac{8fLQ^2}{\pi^2gd^5} = \frac{8f(185)(7/3600)^2}{\pi^2(9.81)d^5} = \frac{5.78\text{E-}5f}{d^5}, \quad \text{or} \quad d \approx 0.0591f^{1/5}$$

Guess $f \approx 0.03$, $d = 0.0591(0.03)^{1/5} \approx 0.0293 \text{ m}$, $Re = \frac{4\rho Q}{\pi\mu d} \approx 84300$, $\frac{\varepsilon}{d} \approx 0.00887$

Iterate: $f_{\text{better}} \approx 0.0372$, $d_{\text{better}} \approx 0.0306 \text{ m}$, $Re_{\text{better}} \approx 80700$, $\varepsilon/d|_{\text{better}} \approx 0.00850$, etc. The process converges to $f \approx 0.0367$, $d \approx 0.0305 \text{ m}$. *Ans.*

P6.93 In Moody's Example 6.6, the 6-inch diameter, 200-ft-long asphalted cast iron pipe has a pressure drop of about 280 lbf/ft^2 when the average water velocity is 6 ft/s . Compare this to an *annular* cast iron pipe with an inner diameter of 6 in and the same annular average velocity of 6 ft/s . (a) What outer diameter would cause the flow to have the same pressure drop of 280 lbf/ft^2 ? (b) How do the cross-section areas compare, and why? Use the hydraulic diameter approximation.

Solution: Recall the Ex. 6.6 data, $\varepsilon = 0.0004$ ft. For water at 68°F, take $\rho = 1.94$ slug/ft³ and $\mu = 2.09E-5$ slug/ft-sec. The hydraulic diameter of an annulus is $D_h = 2(R_o - R_i)$, where $R_i = 0.25$ ft. We know the pressure drop, hence the head loss is

$$h_f = f \frac{L}{D_h} \frac{V^2}{2g} = f \frac{200 \text{ ft}}{2(R_o - 0.25 \text{ ft})} \frac{(6 \text{ ft/s})^2}{32.2 \text{ ft/s}^2} = \frac{\Delta p}{\rho g} = \frac{280 \text{ lbf/ft}^2}{62.4 \text{ lbf/ft}^3} = 4.49 \text{ ft}$$

We do not know f or R_o . The additional relation is the Moody friction factor correlation:

$$\frac{1}{\sqrt{f}} \approx -2.0 \log_{10} \left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}_{D_h} \sqrt{f}} \right) \quad \text{where} \quad \text{Re}_{D_h} = \frac{\rho V D_h}{\mu} = \frac{(1.94)(6.0)[2(R_o - 0.25)]}{2.09E-5}$$

(a) For $\varepsilon = 0.0004$ ft, solve these two simultaneously, using EES or Excel, to obtain

$$f = 0.0199 ; \text{Re}_{D_h} = 276,000 ; R_o = \mathbf{0.498 \text{ ft}} \quad \text{Ans.}(a)$$

(b) The annular gap is $0.498 - 0.25 = 0.248$ ft, just about equal to the inner radius. However, the annular area is **three times the area of Moody's pipe!** *Ans.* (b) The annular pipe has much more wall area than a hollow pipe, more friction, so more area is needed to match the pressure drop.

6.109 In Fig. P6.109 there are 125 ft of 2-in pipe, 75 ft of 6-in pipe, and 150 ft of 3-in pipe, all cast iron. There are three 90° elbows and an open globe valve, all flanged. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 0.16 ft³/s of water at 20°C?

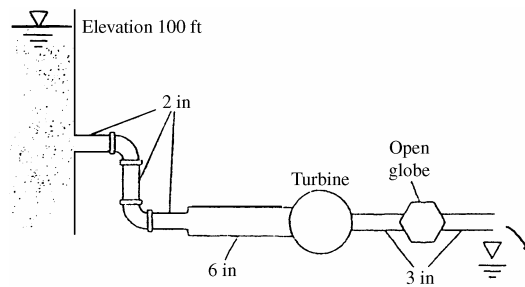


Fig. P6.109

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft³ and $\mu = 2.09\text{E-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. The 2", 6", and 3" pipes have, respectively,

- (a) $L/d = 750$, $\varepsilon/d = 0.0051$; (b) $L/d = 150$, $\varepsilon/d = 0.0017$;
 (c) $L/d = 600$, $\varepsilon/d = 0.0034$

The flow rate is known, so each velocity, Reynolds number, and f can be calculated:

$$V_a = \frac{0.16}{\pi(2/12)^2/4} = 7.33 \frac{\text{ft}}{\text{s}}; \quad \text{Re}_a = \frac{1.94(7.33)(2/12)}{2.09\text{E-}5} = 113500, \quad f_a \approx 0.0314$$

$$\text{Also, } V_b = 0.82 \text{ ft/s, } \text{Re}_b = 37800, \quad f_c \approx 0.0266; \quad V_c = 3.26, \quad \text{Re}_c = 75600, \quad f_c \approx 0.0287$$

Finally, the minor loss coefficients may be tabulated:

$$\begin{aligned} \text{sharp 2" entrance: } K &= 0.5; & \text{three 2" 90° elbows: } K &= 3(0.95) \\ \text{2" sudden expansion: } K &\approx 0.79; & \text{3" open globe valve: } K &\approx 6.3 \end{aligned}$$

The turbine head equals the elevation difference minus losses and the exit velocity head:

$$\begin{aligned} h_t &= \Delta z - \sum h_f - \sum h_m - V_c^2/(2g) \\ &= 100 - \frac{(7.33)^2}{2(32.2)} [0.0314(750) + 0.5 + 3(0.95) + 0.79] \\ &\quad - \frac{(0.82)^2}{2(32.2)} (0.0266)(150) - \frac{(3.26)^2}{2(32.2)} [0.0287(600) + 6.3 + 1] \approx \mathbf{72.8 \text{ ft}} \end{aligned}$$

The resulting turbine power = $\rho g Q h_t = (62.4)(0.16)(72.8) \div 550 \approx \mathbf{1.32 \text{ hp}}$. *Ans.*

6.113 The parallel galvanized-iron pipe system of Fig. P6.113 delivers water at 20°C with a total flow rate of 0.036 m³/s. If the pump is wide open and not running, with a loss coefficient $K = 1.5$, determine (a) the flow rate in each pipe and (b) the overall pressure drop.

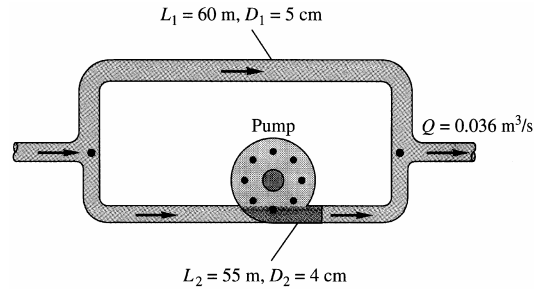


Fig. P6.113

Solution: For water at 20°C, take $\rho = 998 \text{ kg/m}^3$ and $\mu = 0.001 \text{ kg/m}\cdot\text{s}$. For galvanized iron, $\varepsilon = 0.15 \text{ mm}$. Assume turbulent flow, with Δp the same for each leg:

$$h_{f1} = f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g} = h_{f2} + h_{m2} = \frac{V_2^2}{2g} \left(f_2 \frac{L_2}{d_2} + 1.5 \right),$$

$$\text{and } Q_1 + Q_2 = (\pi/4)d_1^2 V_1 + (\pi/4)d_2^2 V_2 = Q_{\text{total}} = 0.036 \text{ m}^3/\text{s}$$

When the friction factors are correctly found from the Moody chart, these two equations may be solved for the two velocities (or flow rates). Begin by guessing $f \approx 0.020$:

$$(0.02) \left(\frac{60}{0.05} \right) \frac{V_1^2}{2(9.81)} = \frac{V_2^2}{2(9.81)} \left[(0.02) \left(\frac{55}{0.04} \right) + 1.5 \right], \text{ solve for } V_1 \approx 1.10V_2$$

$$\text{then } \frac{\pi}{4}(0.05)^2(1.10V_2) + \frac{\pi}{4}(0.04)^2 V_2 = 0.036. \text{ Solve } V_2 \approx 10.54 \frac{\text{m}}{\text{s}}, V_1 \approx 11.59 \frac{\text{m}}{\text{s}}$$

$$\text{Correct } Re_1 \approx 578000, f_1 \approx 0.0264, Re_2 \approx 421000, f_2 \approx 0.0282, \text{ repeat.}$$

The 2nd iteration converges: $f_1 \approx 0.0264$, $V_1 = 11.69 \text{ m/s}$, $f_2 \approx 0.0282$, $V_2 = 10.37 \text{ m/s}$,

$$Q_1 = A_1 V_1 = \mathbf{0.023 \text{ m}^3/\text{s}}, \quad Q_2 = A_2 V_2 = \mathbf{0.013 \text{ m}^3/\text{s}}. \quad \text{Ans. (a)}$$

The pressure drop is the same in either leg:

$$\Delta p = f_1 \frac{L_1}{d_1} \frac{\rho V_1^2}{2} = \left(f_2 \frac{L_2}{d_2} + 1.5 \right) \frac{\rho V_2^2}{2} \approx \mathbf{2.16E6 \text{ Pa}} \quad \text{Ans. (b)}$$

C6.4 Suppose you build a house out in the ‘boonies,’ where you need to run a pipe to the nearest water supply, which fortunately is about 1 km above the elevation of your house. The gage pressure at the water supply is 1 MPa. You require a minimum of 3 gal/min when your end of the pipe is open to the atmosphere. To minimize cost, you want to buy the smallest possible diameter pipe with an extremely smooth surface.

(a) Find the total head loss from pipe inlet to exit, neglecting minor losses.

- (b) Which is more important to this problem, the head loss due to elevation difference, or the head loss due to pressure drop in the pipe?
 (c) Find the minimum required pipe diameter.

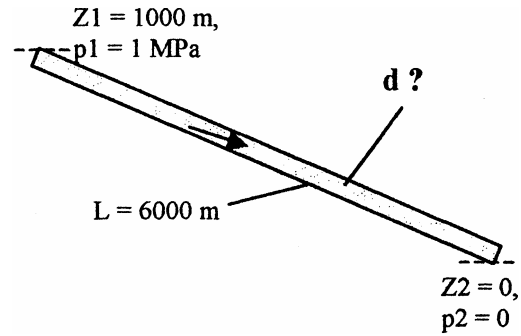


Fig. C6.4

Solution: Convert 3.0 gal/min to $1.89\text{E-}4 \text{ m}^3/\text{s}$. Let 1 be the inlet and 2 be the outlet and write the steady-flow energy equation:

$$\frac{p_{1\text{gage}}}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_{2\text{gage}}}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f$$

$$\text{or: } h_f = z_1 - z_2 + \frac{p_{1\text{gage}}}{\rho g} = 1000 \text{ m} + \frac{1\text{E}6 \text{ kPa}}{998(9.81)} = 1000 + 102 = 1102 \text{ m} \quad \text{Ans. (a)}$$

- (b) Thus, *elevation drop* of 1000 m is more important to head loss than $\Delta p/\rho g = 102 \text{ m}$.
 (c) To find the minimum diameter, iterate between flow rate and the Moody chart:

$$h_f = f \frac{L V^2}{d 2g}, \quad L = 6000 \text{ m}, \quad \frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{\text{Re} \sqrt{f}} \right), \quad V = \frac{Q}{\pi d^2/4},$$

$$Q = 1.89\text{E-}4 \frac{\text{m}^3}{\text{s}}, \quad \text{Re} = \frac{Vd}{\nu}$$

We are given $h_f = 1102 \text{ m}$ and $\nu_{\text{water}} = 1.005\text{E-}6 \text{ m}^2/\text{s}$. We can iterate, if necessary, or use **EES**, which can swiftly arrive at the final result:

$$f_{\text{smooth}} = 0.0266; \quad \text{Re} = 17924; \quad V = 1.346 \text{ m/s}; \quad \mathbf{d_{\min} = 0.0134 \text{ m}} \quad \text{Ans. (c)}$$