5.35 The torque M required to turn the cone-plate viscometer in Fig. P5.35 depends upon the radius R, rotation rate Ω, fluid viscosity μ, and cone angle θ. Rewrite this relation in dimensionless form. How does the relation simplify if it is known that M is proportional to θ?

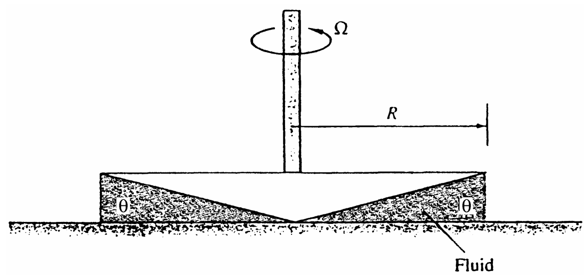


Fig. P5.35

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} M & = \text{fcn}(& R, & \Omega, & \mu, & \theta) \\ \{ML^2/T^2\} & & \{L\} & \{1/T\} & \{M/LT\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups, capable of only one reasonable arrangement, as follows:

$$\frac{\mathbf{M}}{\boldsymbol{\mu\Omega}\mathbf{R}^3} = \mathbf{fcn}(\boldsymbol{\theta}); \quad \text{if } M \propto \theta, \quad \text{then } \frac{\mathbf{M}}{\boldsymbol{\mu\Omega\theta}\mathbf{R}^3} = \mathbf{constant} \quad \textit{Ans.}$$

See Prob. 1.56 of this Manual, for an analytical solution.

5.47 The differential equation for small-amplitude vibrations $y(x, t)$ of a simple beam is given by

$$\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = 0$$

where ρ = beam material density
A = cross-sectional area
I = area moment of inertia
E = Young's modulus

Use only the quantities ρ, E, and A to nondimensionalize y, x, and t, and rewrite the differential equation in dimensionless form. Do any parameters remain? Could they be removed by further manipulation of the variables?

Solution: The appropriate dimensionless variables are

$$y^* = \frac{y}{\sqrt{A}}; \quad t^* = t\sqrt{\frac{E}{\rho A}}; \quad x^* = \frac{x}{\sqrt{A}}$$

Substitution into the PDE above yields a dimensionless equation with *one* parameter:

$$\frac{\partial^2 \mathbf{y}^*}{\partial \mathbf{t}^{*2}} + \left(\frac{\mathbf{I}}{\mathbf{A}^2}\right)\frac{\partial^4 \mathbf{y}^*}{\partial \mathbf{x}^{*4}} = \mathbf{0}; \quad \text{One geometric parameter: } \frac{\mathbf{I}}{\mathbf{A}^2} \quad \textit{Ans.}$$

We could *remove* (I/A^2) completely by redefining $\mathbf{x^* = x/I^{1/4}}$. *Ans.*

5.71 The pressure drop in a venturi meter (Fig. P3.165) varies only with the fluid density, pipe approach velocity, and diameter ratio of the meter. A model venturi meter tested in water at 20°C shows a 5-kPa drop when the approach velocity is 4 m/s. A geometrically similar prototype meter is used to measure gasoline at 20°C and a flow rate of 9 m^3/min. If the prototype pressure gage is most accurate at 15 kPa, what should the upstream pipe diameter be?

Solution: Given Δp = fcn(ρ, V, d/D), then by dimensional analysis $\Delta p/(\rho V^2)$ = fcn(d/D). For water at 20°C, take ρ = 998 kg/m^3. For gasoline at 20°C, take ρ = 680 kg/m^3. Then, using the water 'model' data to obtain the function "fcn(d/D)", we calculate

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{5000}{(998)(4.0)^2} = 0.313 = \frac{\Delta p_p}{\rho_p V_p^2} = \frac{15000}{(680)V_p^2}, \quad \text{solve for } V_p \approx 8.39\ \frac{m}{s}$$

$$\text{Given} \quad Q = \frac{9}{60}\ \frac{m^3}{s} = V_p A_p = (8.39)\frac{\pi}{4}D_p^2, \quad \text{solve for best } \mathbf{D_p \approx 0.151\ m} \quad \textit{Ans.}$$

5.82 A prototype ship is 400 ft long and has a wetted area of 30,000 ft^2. A one-eightieth-scale model is tested in a tow tank according to Froude scaling at speeds of 1.3, 2.0, and 2.7 kn (1 kn = 1.689 ft/s). The measured friction drag of the model at these speeds is 0.11, 0.24, and 0.41 lbf, respectively. What are the three prototype speeds? What is the estimated prototype friction drag at these speeds if we correct for Reynolds-number discrepancy by extrapolation?

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3, μ = 2.09E–5 slug/ft·s. Convert the velocities to ft/sec. Calculate the Reynolds numbers for the model data:

V_m, ft/s:	2.19	3.38	4.56
$Re_m = \rho VL/\mu$:	1.02E6	1.57E6	2.12E6
$C_{Fm} = F/\rho V^2L^2$:	0.000473	0.000433	0.000407

The data may be fit to the Power-law expression $C_{Fm} \approx \mathbf{0.00805/Re^{0.205}}$. The related *prototype* speeds are given by Froude scaling, $V_p = V_m/\sqrt{\alpha}$, where $\alpha = 1/80$:

V_m, ft/s:	2.19	3.38	4.56	
V_p, ft/s:	**19.6**	**30.2**	**40.8**	*Ans.* (a)

Then we may compute the prototype Reynolds numbers and friction drag coefficients:

$Re_p = \rho VL/\mu$:	7.27E8	1.12E9	1.51E9

Estimate the friction-drag coefficients by extrapolating the Power-law fit listed previously:

$C_{Fp} = F/\rho V^2L^2$:	0.000123	0.000112	0.000106	
$F_p = C_{Fp}\rho V_p^2 L_p^2$:	**14600 lbf**	**31800 lbf**	**54600 lbf**	*Ans.* (b)

Among other approximations, this extrapolation assumes very smooth surfaces.

5.84 A prototype ocean-platform piling is expected to encounter currents of 150 cm/s and waves of 12-s period and 3-m height. If a one-fifteenth-scale model is tested in a wave channel, what current speed, wave period, and wave height should be encountered by the model?

Solution: Given $\alpha = 1/15$, apply straight Froude scaling (Fig. 5.6*b*) to these results:

$$\text{Velocity:}\quad V_m = V_p\sqrt{\alpha} = \frac{150}{\sqrt{15}} = \mathbf{39\ \frac{cm}{s}}$$

$$\text{Period:}\quad T_m = T_p\sqrt{\alpha} = \frac{12}{\sqrt{15}} = \mathbf{3.1\ s};\quad \text{Height:}\quad H_m = \alpha H_p = \frac{3}{15} = \mathbf{0.20\ m}\quad \textit{Ans.}$$

C5.4 The Taco Inc. Model 4013 centrifugal pump has an impeller of diameter D = 12.95 in. When pumping 20°C water at Ω = 1160 rev/min, the measured flow rate Q and pressure rise Δp are given by the manufacturer as follows:

Q (gal/min)	~	200	300	400	500	600	700
Δp (psi)	~	36	35	34	32	29	23

(a) Assuming that Δp = fcn(ρ, Q, D, Ω), use the Pi theorem to rewrite this function in terms of dimensionless parameters and then plot the given data in dimensionless form. (b) It is desired to use the same pump, running at 900 rev/min, to pump 20°C gasoline at 400 gal/min. According to your dimensionless correlation, what pressure rise Δp is expected, in lbf/in^2?

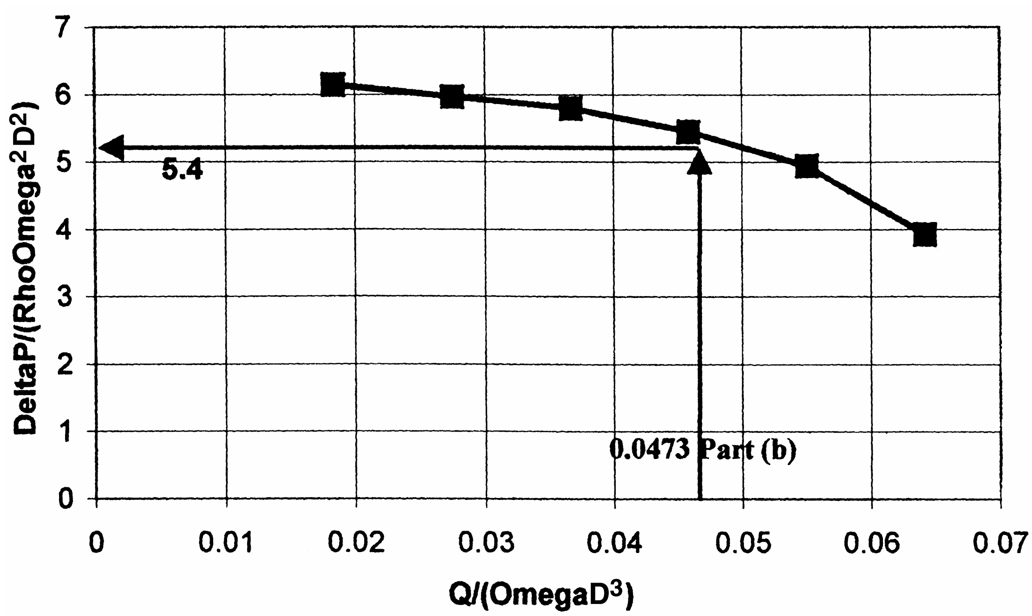
Solution: There are n = 5 variables and j = 3 dimensions (M, L, T), hence we expect $n - j = 5 - 3 =$ **2** Pi groups. The author selects (ρ, D, Ω) as repeating variables, whence

$$\Pi_1 = \frac{\Delta p}{\rho\Omega^2 D^2};\quad \Pi_2 = \frac{Q}{\Omega D^3},\quad or:\quad \boldsymbol{\frac{\Delta p}{\rho\Omega^2 D^2} = fcn\left(\frac{Q}{\Omega D^3}\right)}\quad \textit{Ans. (a)}$$

Convert the data to this form, using Ω = 19.33 rev/s, D = 1.079 ft, ρ = 1.94 slug/ft^3, and use Δp in lbf/ft^2, not psi, and Q in $\text{ft}^3\text{/s}$, not gal/min:

Q (gal/min)	~	200	300	400	500	600	700
$\Delta p/(\rho\Omega^2 D^2)$:		*6.14*	*5.97*	*5.80*	*5.46*	*4.95*	*3.92*
$Q/(\Omega D^3)$:		*0.0183*	*0.0275*	*0.0367*	*0.0458*	*0.0550*	*0.0642*

The dimensionless plot of Π1 versus Π2 is shown below.



(b) The dimensionless chart above is valid for the new conditions, also. Convert 400 gal/min to 0.891 ft^3/s and $\Omega = 900$ rev/min to 15 rev/s. Then evaluate Π2:

$$\Pi_2 = \frac{Q}{\Omega D^3} = \frac{0.891}{15(1.079)^3} = \mathbf{0.0473}$$

This value is entered in the chart above, from which we see that the corresponding value of Π1 is about **5.4**. For gasoline (Table A-3), ρ = 1.32 $slug/ft^3$. Then this new running condition with gasoline corresponds to

$$\Pi_2 = 5.4 = \frac{\Delta p}{\rho\Omega^2 D^2} = \frac{\Delta p}{1.32(15)^2(1.079)^2}, \quad \textit{solve for } \Delta p = 1870\ \frac{lbf}{ft^2} = \mathbf{13}\ \frac{\mathbf{lbf}}{\mathbf{in^2}} \quad \textit{Ans.}\ \text{(b)}$$