3.17 Incompressible steady flow in the inlet between parallel plates in Fig. P3.17 is uniform, $u = U_o = 8$ cm/s, while downstream the flow develops into the parabolic laminar profile $u = az(z_o - z)$, where a is a constant. If $z_o = 4$ cm and the fluid is SAE 30 oil at 20°C, what is the value of u_{max} in cm/s?

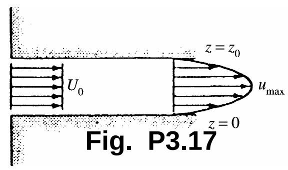


Fig. P3.17

Solution: Let b be the plate width into the paper. Let the control volume enclose the inlet and outlet. The walls are solid, so no flow through the wall. For incompressible flow,

$$0 = Q_{out} - Q_{in} = \int_0^{z_o} az(z_o - z)b\,dz - \int_0^{z_o} U_o b\,dz = abz_o^3/6 - U_o b z_o = 0, \quad \text{or:} \quad a = 6U_o/z_o^2$$

Thus continuity forces the constant a to have a particular value. Meanwhile, a is also related to the maximum velocity, which occurs at the center of the parabolic profile:

$$\text{At } z = z_o/2: \quad u = u_{max} = a\left(\frac{z_o}{2}\right)\left(z_o - \frac{z_o}{2}\right) = az_o^2/4 = (6U_o/z_o^2)(z_o^2/4)$$

$$\text{or:} \quad u_{max} = \frac{3}{2}U_o = \frac{3}{2}(8\text{ cm/s}) = \mathbf{12\ \frac{cm}{s}} \quad \textit{Ans.}$$

Note that the result is independent of z_o or of the particular fluid, which is SAE 30 oil.

3.33 In some wind tunnels the test section is perforated to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig. P3.33 contains 1200 holes of 5-mm diameter each per square meter of wall area. The suction velocity through each hole is $V_r = 8$ m/s, and the test-section entrance velocity is $V_1 = 35$ m/s. Assuming incompressible steady flow of air at 20°C, compute (a) V_o, (b) V_2, and (c) V_f, in m/s.

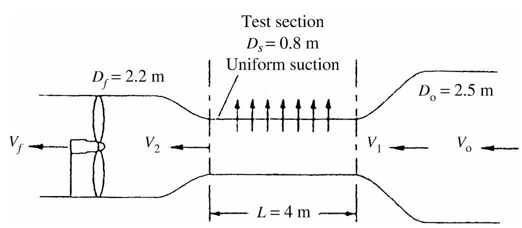


Fig. P3.33

Solution: The test section wall area is $(\pi)(0.8\text{ m})(4\text{ m}) = 10.053\text{ m}^2$, hence the total number of holes is (1200)(10.053) = 12064 holes. The total suction flow leaving is

$$Q_{suction} = NQ_{hole} = (12064)(\pi/4)(0.005\text{ m})^2(8\text{ m/s}) \approx 1.895\text{ m}^3/\text{s}$$

$$\text{(a)} \quad \text{Find } V_o: \quad Q_o = Q_1 \quad \text{or} \quad V_o\frac{\pi}{4}(2.5)^2 = (35)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for} \quad V_o \approx \mathbf{3.58\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

(b) $Q_2 = Q_1 - Q_{suction} = (35)\frac{\pi}{4}(0.8)^2 - 1.895 = V_2\frac{\pi}{4}(0.8)^2,$

$$\text{or:}\quad V_2 \approx \mathbf{31.2\ \frac{m}{s}}\quad \textit{Ans.}\ \text{(b)}$$

(c) Find V_f: $Q_f = Q_2$ or $V_f\frac{\pi}{4}(2.2)^2 = (31.2)\frac{\pi}{4}(0.8)^2,$

$$\text{solve for}\quad V_f \approx \mathbf{4.13\ \frac{m}{s}}\quad \textit{Ans.}\ \text{(c)}$$

3.44 Consider uniform flow past a cylinder with a V-shaped *wake*, as shown. Pressures at (1) and (2) are equal. Let *b* be the width into the paper. Find a formula for the force *F* on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 Lb)$.

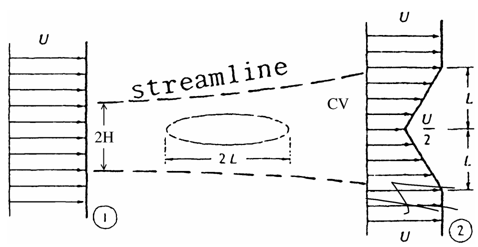


Fig. P3.44

Solution: The proper CV is the entrance (1) and exit (2) plus *streamlines* above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

$$0 = \int_2 \rho u\, dA - \int_1 \rho u\, dA = 2\int_0^L \rho\frac{U}{2}\left(1+\frac{y}{L}\right)b\, dy - 2\rho UbH,$$

where 2H is the inlet height. Solve for H = 3L/4.

Now the linear momentum relation is used. Note that the drag force F is to the right (force of the fluid on the body) thus the force F *of the body on fluid is to the left*. We obtain,

$$\sum F_x = 0 = \int_2 u\rho u\, dA - \int_1 u\rho u\, dA = 2\int_0^L \frac{U}{2}\left(1+\frac{y}{L}\right)\rho\frac{U}{2}\left(1+\frac{y}{L}\right)b\, dy - 2H\rho U^2 b = -F_{drag}$$

$$\text{Use}\quad H = \frac{3L}{4},\quad \text{then}\quad F_{drag} = \frac{3}{2}\rho U^2 Lb - \frac{7}{6}\rho U^2 Lb \approx \mathbf{\frac{1}{3}\rho U^2 Lb}\quad \textit{Ans.}$$

The dimensionless force, or drag coefficient $F/(\rho U^2 Lb)$, equals $\mathbf{C_D = 1/3}$. *Ans.*

3.46 When a jet strikes an inclined plate, it breaks into two jets of equal velocity V but unequal fluxes αQ at (2) and $(1 - \alpha)$Q at (3), as shown. Find α, assuming that the tangential force on the plate is zero. Why doesn't the result depend upon the properties of the jet flow?

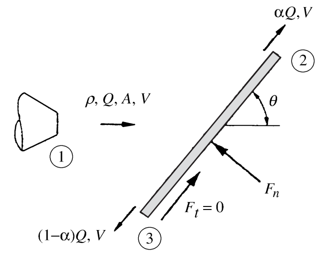


Fig. P3.46

Solution: Let the CV enclose all three jets and the surface of the plate. Analyze the force and momentum balance *tangential* to the plate:

$$\sum F_t = F_t = 0 = \dot{m}_2 V + \dot{m}_3(-V) - \dot{m}_1 V\cos\theta$$

$$= \alpha\dot{m}V - (1-\alpha)\dot{m}V - \dot{m}V\cos\theta = 0, \quad \text{solve for } \alpha = \mathbf{\frac{1}{2}(1+\cos\theta)} \quad \textit{Ans.}$$

The jet mass flow cancels out. Jet (3) has a fractional flow $(1 - \alpha) = (1/2)(1 - \cos\theta)$.

3.102 As can often be seen in a kitchen sink when the faucet is running, a high-speed channel flow (V_1, h_1) may "jump" to a low-speed, low-energy condition (V_2, h_2) as in Fig. P3.102. The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Use the continuity and momentum relations to find h_2 and V_2 in terms of (h_1, V_1).

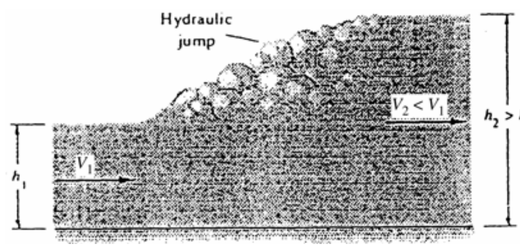


Fig. P3.102

Solution: The CV cuts through sections 1 and 2 and surrounds the jump, as shown. Wall shear is neglected. There are no obstacles. The only forces are due to hydrostatic pressure:

$$\sum F_x = 0 = \frac{1}{2}\rho g h_1(h_1 b) - \frac{1}{2}\rho g h_2(h_2 b) = \dot{m}(V_2 - V_1),$$

$$\text{where } \dot{m} = \rho V_1 h_1 b = \rho V_2 h_2 b$$

$$\text{Solve for} \quad V_2 = \mathbf{V_1 h_1/h_2} \quad \text{and} \quad h_2/h_1 = \mathbf{-\frac{1}{2} + \frac{1}{2}\sqrt{1+8V_1^2/(gh_1)}} \quad \textit{Ans.}$$

3.114 The 3-arm lawn sprinkler of Fig. P3.114 receives 20°C water through the center at 2.7 m^3/hr. If collar friction is neglected, what is the steady rotation rate in rev/min for (a) $\theta = 0°$; (b) $\theta = 40°$?

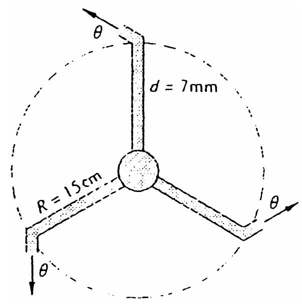


Fig. P3.114

Solution: The velocity exiting each arm is

$$V_o = \frac{Q/3}{(\pi/4)d^2} = \frac{2.7/[(3600)(3)]}{(\pi/4)(0.007)^2} = 6.50\ \frac{m}{s}$$

With negligible air drag and bearing friction, the steady rotation rate (Example 3.15) is

$$\omega_{final} = \frac{V_o \cos\theta}{R} \quad \text{(a) } \theta = 0°: \quad \omega = \frac{(6.50)\cos 0°}{0.15\text{ m}} = 43.3\ \frac{rad}{s} = \mathbf{414}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) } \theta = 40°: \quad \omega = \omega_o \cos\theta = (414)\cos 40° = \mathbf{317}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans.}\text{ (b)}$$

3.137 A fireboat draws seawater (SG = 1.025) from a submerged pipe and discharges it through a nozzle, as in Fig. P3.137. The total head loss is 6.5 ft. If the pump efficiency is 75 percent, what horsepower motor is required to drive it?

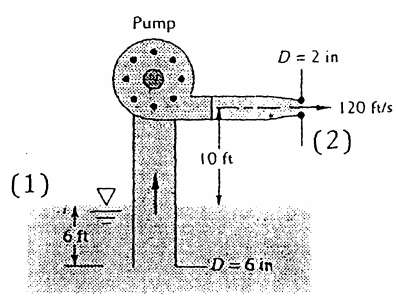


Fig. P3.137

Solution: For seawater, $\gamma = 1.025(62.4) = 63.96$ lbf/ft^3. The energy equation becomes

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 0 = 0 + \frac{(120)^2}{2(32.2)} + 10 + 6.5 - h_p$$

Solve for $h_p = 240$ ft. The flow rate is $Q = V_2A_2 = (120)(\pi/4)(2/12)^2 = 2.62\ ft^3/s$. Then

$$P_{pump} = \frac{\gamma Q h_p}{\text{efficiency}} = \frac{(63.96)(2.62)(240)}{0.75} = 53600\ \frac{ft\cdot lbf}{s} \approx \mathbf{97\ hp} \quad \textit{Ans.}$$

3.144 The pump in Fig. P3.144 creates a 20°C water jet oriented to travel a maximum horizontal distance. System friction head losses are 6.5 m. The jet may be approximated by the trajectory of frictionless particles. What power must be delivered by the pump?

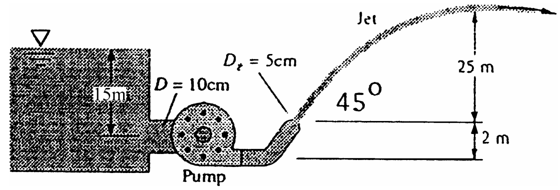


Fig. P3.144

Solution: For maximum travel, the jet must exit at $\theta = 45°$, and the exit velocity must be

$$V_2 \sin\theta = \sqrt{2g\Delta z_{max}} \quad \text{or:} \quad V_2 = \frac{[2(9.81)(25)]^{1/2}}{\sin 45°} \approx 31.32\ \frac{m}{s}$$

The steady flow energy equation for the piping system may then be evaluated:

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 15 = 0 + (31.32)^2/[2(9.81)] + 2 + 6.5 - h_p, \quad \text{solve for } h_p \approx 43.5\ \text{m}$$

$$\text{Then} \quad P_{pump} = \gamma Q h_p = (9790)\left[\frac{\pi}{4}(0.05)^2(31.32)\right](43.5) \approx \mathbf{26200\ W} \quad \textit{Ans.}$$

3.160 The air-cushion vehicle in Fig. P3.160 brings in sea-level standard air through a fan and discharges it at high velocity through an annular skirt of 3-cm clearance. If the vehicle weighs 50 kN, estimate (a) the required airflow rate and (b) the fan power in kW.

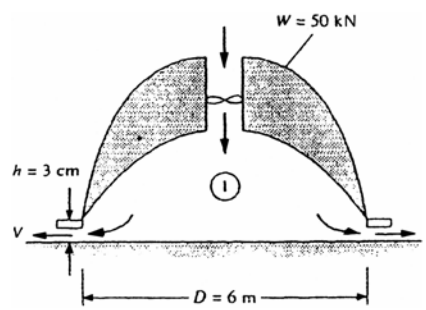


Fig. P3.160

Solution: The air inside at section 1 is nearly stagnant (V ≈ 0) and supports the weight and also drives the flow out of the interior into the atmosphere:

$$p_1 \approx p_{o1}: \quad p_{o1} - p_{atm} = \frac{\text{weight}}{\text{area}} = \frac{50{,}000\ \text{N}}{\pi(3\ \text{m})^2} = \frac{1}{2}\rho V_{exit}^2 = \frac{1}{2}(1.205)V_{exit}^2 \approx 1768\ \text{Pa}$$

$$\text{Solve for} \quad V_{exit} \approx 54.2\ \text{m/s, whence} \quad Q_e = A_e V_e = \pi(6)(0.03)(54.2) = 30.6\ \frac{m^3}{s}$$

Then the power required by the fan is $P = Q_e \Delta p = (30.6)(1768) \approx$ **54000 W** *Ans.*

3.174 In Fig. P3.174 the piston drives water at 20°C. Neglecting losses, estimate the exit velocity *V*2 ft/s. If *D*2 is further constricted, what is the maximum possible value of *V*2?

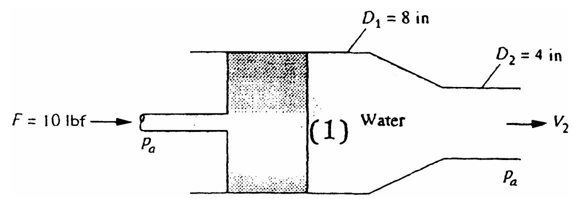


Fig. P3.174

Solution: Find p1 from a freebody of the piston:

$$\sum F_x = F + p_a A_1 - p_1 A_1, \quad \text{or:} \quad p_1 - p_a = \frac{10.0 \text{ lbf}}{(\pi/4)(8/12)^2} \approx 28.65 \ \frac{\text{lbf}}{\text{ft}^2}$$

Now apply continuity and Bernoulli from 1 to 2:

$$V_1 A_1 = V_2 A_2, \quad \text{or} \quad V_1 = \frac{1}{4} V_2; \quad \frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_a}{\rho} + \frac{V_2^2}{2}$$

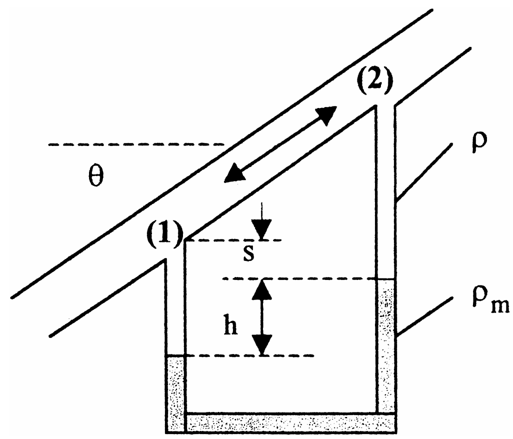
Introduce $p_1 - p_a$ and substitute for V_1 to obtain $V_2^2 = \dfrac{2(28.65)}{1.94(1-1/16)}$,

$$V_2 = \mathbf{5.61} \ \frac{\mathbf{ft}}{\mathbf{s}} \quad \textit{Ans.}$$

If we reduce section 2 to a pinhole, V2 will drop off slowly until V1 vanishes:

$$\text{Severely constricted section 2:} \quad V_2 = \sqrt{\frac{2(28.65)}{1.94(1-0)}} \approx \mathbf{5.43} \ \frac{\mathbf{ft}}{\mathbf{s}} \quad \textit{Ans.}$$

C3.1 In a certain industrial process, oil of density ρ flows through the inclined pipe in the figure. A U-tube manometer with fluid density ρ_m, measures the pressure difference between points 1 and 2, as shown. The flow is steady, so that fluids in the U-tube are stationary. (a) Find an analytic expression for $p_1 - p_2$ in terms of system parameters. (b) Discuss the conditions on *h* necessary for there to be no flow in the pipe. (c) What about flow *up*, from 1 to 2? (d) What about flow *down*, from 2 to 1?



Solution: (a) Start at 1 and work your way around the U-tube to point 2:

$$p_1 + \rho gs + \rho gh - \rho_m gh - \rho gs - \rho g\Delta z = p_2,$$

$$or: \quad \mathbf{p_1 - p_2 = \rho g\Delta z + (\rho_m - \rho)gh} \quad \text{where } \Delta z = z_2 - z_1 \quad \textit{Ans. (a)}$$

(b) If there is no flow, the pressure is entirely hydrostatic, therefore $\Delta p = \rho g$ and, since $\rho_m \neq \rho$, it follows from Ans. (a) above that **h = 0** *Ans.* (b)

(c) If *h* is positive (as in the figure above), p_1 is greater than it would be for no flow, because of head losses in the pipe. **Thus, if h > 0, flow is *up* from 1 to 2**. *Ans.* (c)

(d) If *h* is negative, p_1 is less than it would be for no flow, because the head losses act against hydrostatics. **Thus, if h < 0, flow is *down* from 2 to 1**. *Ans.* (d)

Note that *h* is a direct measure of flow, regardless of the angle θ of the pipe.