2.42 Small pressure differences can be measured by the two-fluid manometer in Fig. P2.42, where ρ_2 is only slightly larger than ρ_1. Derive a formula for $p_A - p_B$ if the reservoirs are very large.

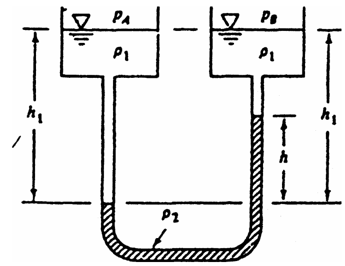


Fig. P2.42

Solution: Apply the hydrostatic formula from A to B:

$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h) = p_B$$

$$\text{Solve for} \quad \mathbf{p_A - p_B = (\rho_2 - \rho_1)gh} \quad \textit{Ans.}$$

If $(\rho_2 - \rho_1)$ is very small, *h* will be very large for a given Δp (a sensitive manometer).

2.71 In Fig. P2.71 gate AB is 3 m wide into the paper and is connected by a rod and pulley to a concrete sphere (SG = 2.40). What sphere diameter is just right to close the gate?

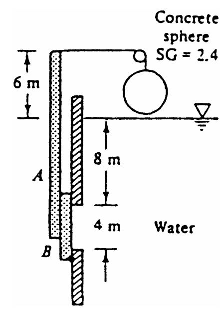


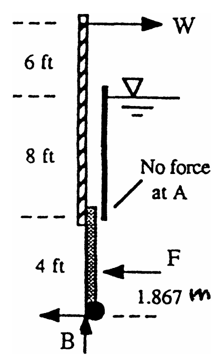
Fig. P2.71

Solution: The centroid of AB is 10 m down from the surface, hence the hydrostatic force is

$$F = \gamma h_{CG} A = (9790)(10)(4 \times 3)$$
$$= 1.175E6 \text{ N}$$

The line of action is slightly below the centroid:

$$y_{CP} = -\frac{(1/12)(3)(4)^3 \sin 90^\circ}{(10)(12)} = -0.133 \text{ m}$$



Sum moments about B in the freebody at right to find the pulley force or weight W:

$$\sum M_B = 0 = W(6 + 8 + 4 \text{ m}) - (1.175E6)(2.0 - 0.133 \text{ m}), \quad \text{or} \quad W = 121800 \text{ N}$$

Set this value equal to the weight of a solid concrete sphere:

$$W = 121800 \text{ N} = \gamma_{concrete}\frac{\pi}{6}D^3 = (2.4)(9790)\frac{\pi}{6}D^3, \quad \text{or:} \quad D_{sphere} = \mathbf{2.15\ m} \quad \textit{Ans.}$$

2.99 A 2-ft-diam sphere weighing 400 kbf closes the 1-ft-diam hole in the tank bottom. Find the force F to dislodge the sphere from the hole.

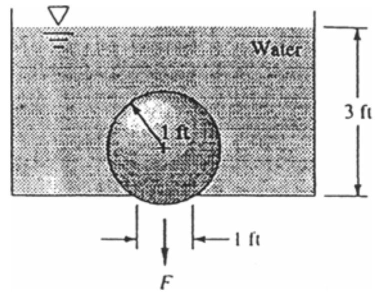
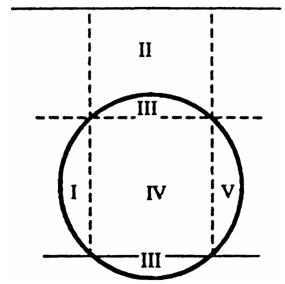


Fig. P2.99

Solution: <u>NOTE: This problem is laborious</u>! Break up the system into regions I,II,III,IV, & V. The respective volumes are:

$$\upsilon_{III} = 0.0539\ \text{ft}^3; \quad \upsilon_{II} = 0.9419\ \text{ft}^3$$
$$\upsilon_{IV} = \upsilon_{I} = \upsilon_{V} = 1.3603\ \text{ft}^3$$



Then the hydrostatic forces are:

$$F_{down} = \gamma\upsilon_{II} = (62.4)(0.9419) = 58.8\ \text{lbf}$$
$$F_{up} = \gamma(\upsilon_I + \upsilon_V) = (62.4)(2.7206) = 169.8\ \text{lbf}$$

Then the required force is $F = W + F_{down} - F_{up} = 400 + 59 - 170 =$ **289 lbf** ↑ *Ans.*

2.121 The uniform beam in the figure is of size L by h by b, with $b,h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

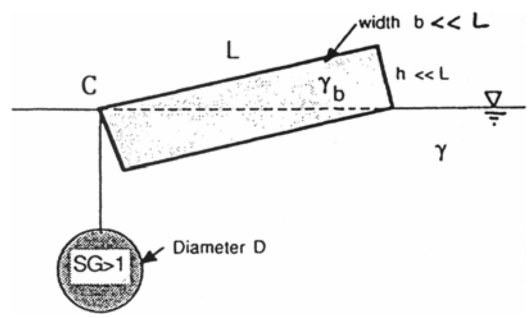


Fig. P2.121

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at L/3 from the left corner. Sum moments about the left corner, point C:

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. (a)}$$

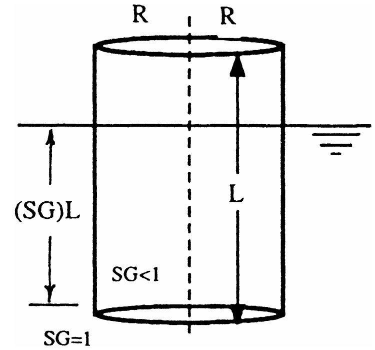
Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

But also $T = (W - B)_{sphere} = (SG - 1)\gamma \dfrac{\pi}{6} D^3$, so that $D = \left[\dfrac{\mathbf{Lhb}}{\boldsymbol{\pi}\mathbf{(SG - 1)}}\right]^{\mathbf{1/3}}$ *Ans.* (b)

2.135 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG, floating in water (SG = 1) with its axis *vertical*. Show that the body is stable if

$$R/L > [2SG(1-SG)]^{1/2}$$



Solution: For a given SG, the body floats with a draft equal to (SG)L, as shown. Its center of gravity G is at L/2 above the bottom. Its center of buoyancy B is at (SG)L/2 above the bottom. Then Eq. (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{\pi R^4/4}{\pi R^2(SG)L} = \frac{R^2}{4(SG)L} = MG + GB = MG + \frac{L}{2} - SG\frac{L}{2}$$

$$\text{Thus}\quad MG > 0 \text{ (stability)}\quad \text{if}\quad \mathbf{R^2/L^2 > 2SG(1-SG)}\quad \textit{Ans.}$$

For example, if SG = 0.8, stability requires that R/L > 0.566.

2.141 The same tank from Prob. 2.139 is now accelerating while rolling *up* a 30° inclined plane, as shown. Assuming rigid-body motion, compute (a) the acceleration **a**, (b) whether the acceleration is up or down, and (c) the pressure at point A if the fluid is mercury at 20°C.

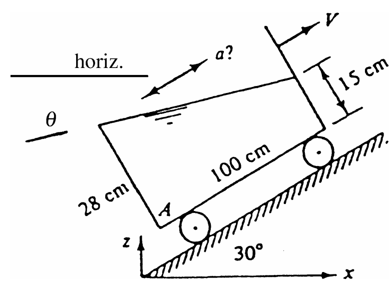


Fig. P2.141

Solution: The free surface is tilted at the angle $\theta = -30° + 7.41° = -22.59°$. This angle must satisfy Eq. (2.55):

$$\tan\theta = \tan(-22.59°) = -0.416 = a_x/(g + a_z)$$

But the 30° incline constrains the acceleration such that $a_x = 0.866a$, $a_z = 0.5a$. Thus

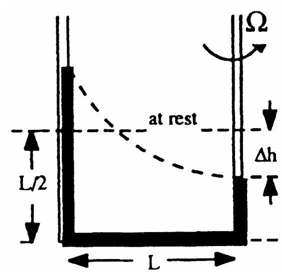
$$\tan\theta = -0.416 = \frac{0.866a}{9.81 + 0.5a},\quad \text{solve for}\quad \mathbf{a \approx -3.80\ \frac{m}{s^2}\ (down)}\quad \textit{Ans. (a, b)}$$

The cartesian components are $a_x = -3.29\ m/s^2$ and $a_z = -1.90\ m/s^2$.
(c) The distance ΔS normal from the surface down to point A is (28 cosθ) cm. Thus

$$p_A = \rho[a_x^2 + (g + a_z)^2]^{1/2} = (13550)[(-3.29)^2 + (9.81 - 1.90)^2]^{1/2}(0.28\cos 7.41°)$$

$$\approx \mathbf{32200\ Pa\ (gage)}\quad \textit{Ans. (c)}$$

2.153 Suppose the U-tube in Prob. 2.150 is not translated but instead is *rotated about the right leg* at 95 r/min. Find the level *h* in the left leg if L = 18 cm and D = 5 mm.



Solution: Convert Ω = 95 r/min = 9.95 rad/s. Then "R" = L = 18 cm, and, since $D \ll L$,

$$\Delta h = \frac{\Omega^2 R^2}{4g} = \frac{(9.95)^2(0.18)^2}{4(9.81)} = 0.082\,m$$

Thus $\quad h_{left\,leg} = 9 + 8.2 = \mathbf{17.2\ cm}$ *Ans.*

1.84* Modify Prob. 1.83 to find the equation of the *pathline* which passes through the point (x_o, y_o) at t = 0. Sketch this pathline.

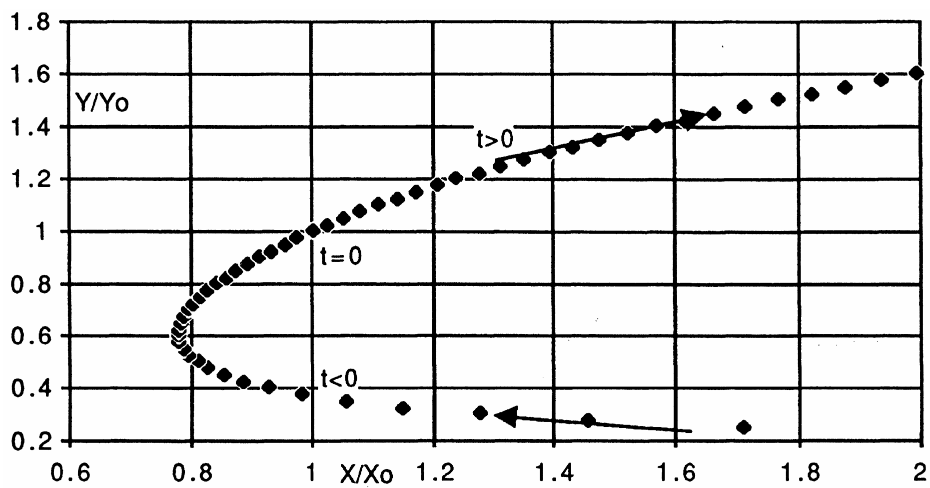
Solution: The pathline is computed by integration, over time, of the velocities:

$$\frac{dx}{dt} = u = x(1+2t), \quad \text{or:} \quad \int \frac{dx}{x} = \int (1+2t)\,dt, \quad \text{or:} \quad x = x_o e^{t+t^2}$$

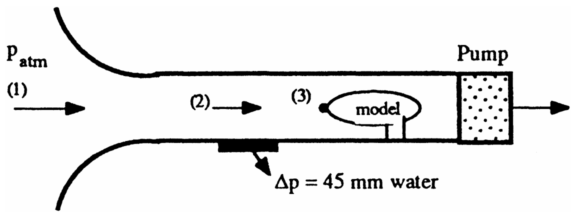
$$\frac{dy}{dt} = v = y, \quad \text{or:} \quad \int \frac{dy}{y} = \int dt, \quad \text{or:} \quad y = y_o e^t$$

We have implemented the initial conditions (x, y) = (x_o, y_o) at t = 0. [We were very lucky, as *planned* for this problem, that u did not depend upon y and v did not depend upon x.] Now eliminate t between these two to get a geometric expression for this particular pathline:

$$\mathbf{x = x_o \exp\{\ln(y/y_o) + \ln^2(y/y_o)\}}$$ This pathline is shown in the sketch below.



3.166 A wind tunnel draws in sea-level standard air from the room and accelerates it into a 1-m by 1-m test section. A pressure transducer in the test section wall measures $\Delta p = 45$ mm water between inside and outside. Estimate (a) the test section velocity in mi/hr; and (b) the absolute pressure at the nose of the model.



Solution: (a) First apply Bernoulli from the atmosphere (1) to (2), using the known Δp:

$$p_a - p_2 = 45 \text{ mm } H_2O = 441 \text{ Pa}; \quad \rho_a = 1.225 \text{ kg/m}^3; \quad p_1 + \frac{\rho}{2}V_1^2 \approx p_2 + \frac{\rho}{2}V_2^2$$

Since $V_1 \approx 0$ and $p_1 = p_a$, we obtain $V_2 = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2(441)}{1.225}} = 26.8 \frac{m}{s} = \mathbf{60}\, \frac{\mathbf{mi}}{\mathbf{hr}}$ *Ans.* (a)

(b) Bernoulli from 1 to 3: both velocities = 0, so $p_{nose} = p_a \approx$ **101350 Pa**. *Ans.* (b)

4.75 Given the following steady *axisymmetric* stream function:

$$\psi = \frac{B}{2}\left(r^2 - \frac{r^4}{2R^2}\right), \quad \text{where } B \text{ and } R \text{ are constants}$$

valid in the region $0 \le r \le R$ and $0 \le z \le L$. (*a*) What are the dimensions of the constant *B*?

(*b*) Show whether this flow possesses a velocity potential and, if so, find it. (*c*) What might this flow represent? [HINT: Examine the axial velocity v_z.]

Solution: (*a*) From the definition of $\psi(r, z)$ in Eqs. (4.105), the dimensions of ψ are $\{L^3/T\}$. Thus *B* has *velocity* dimensions, $\{B\} = \{L/T\}$. *Ans.*(*a*)

(*b*) To test for irrotationality, first find the velocity components from Eqs. (4.106):

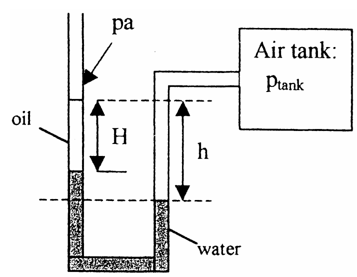
$$v_r = -\frac{1}{r}\frac{\partial \psi}{\partial z} = 0 \quad ; \quad v_z = \frac{1}{r}\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{B}{2}\left(2r - \frac{4r^3}{2R^2}\right) = B\left(1 - \frac{r^2}{R^2}\right)$$

Now evaluate the curl of the velocity, which has only one possible non-zero component. From Appendix D, Eq. (D.11),

$$2\omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} = 0 - \frac{2Br}{R^2} \neq 0$$ **Rotational** , ϕ does not exist. *Ans.*(*b*)

(*c*) The interpretation of the flow follows immediately from the velocity components. The velocity profile is a paraboloid of revolution and represents **Poiseuille pipe flow**, Eq. (4.137). *Ans.*(*c*)

C2.2 A prankster has added oil, of specific gravity SG_o, to the left leg of the manometer at right. Nevertheless, the U-tube is still to be used to measure the pressure in the air tank. (a) Find an expression for *h* as a function of *H* and other parameters in the problem.
(b) Find the special case of your result when $p_{tank} = p_a$.
(c) Suppose H = 5 cm, p_a = 101.2 kPa, SG_o = 0.85, and p_{tank} is 1.82 kPa higher than p_a. Calculate *h* in cm, ignoring surface tension and air density effects.



Solution: Equate pressures at level *i* in the tube:

$$p_i = p_a + \rho g H + \rho_w g(h - H) = p_{tank},$$

$$\rho = SG_o \rho_w \quad \text{(ignore the column of air in the right leg)}$$

Solve for: $$\boldsymbol{h = \frac{p_{tk} - p_a}{\rho_w g} + H(1 - SG_o)}$$ *Ans.* (a)

If $p_{tank} = p_a$, then

$$\boldsymbol{h = H(1 - SG_o)}$$ *Ans.* (b)

(c) For the particular numerical values given above, the answer to (a) becomes

$$h = \frac{1820\ Pa}{998(9.81)} + 0.05(1 - 0.85) = 0.186 + 0.0075 = 0.193\ m = \mathbf{19.3\ cm}$$ *Ans.* (c)

Note that this result is not affected by the actual value of atmospheric pressure.
