1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad \textit{Ans.}$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

1.32 A blimp is approximated by a prolate spheroid 90 m long and 30 m in diameter. Estimate the weight of 20°C gas within the blimp for (a) helium at 1.1 atm; and (b) air at 1.0 atm. What might the difference between these two values represent (Chap. 2)?

Solution: Find a handbook. The volume of a prolate spheroid is, for our data,

$$\upsilon = \frac{2}{3}\pi LR^2 = \frac{2}{3}\pi(90\text{ m})(15\text{ m})^2 \approx 42412\text{ m}^3$$

Estimate, from the ideal-gas law, the respective densities of helium and air:

$$\text{(a)} \quad \rho_{helium} = \frac{p_{He}}{R_{He}T} = \frac{1.1(101350)}{2077(293)} \approx 0.1832\ \frac{kg}{m^3};$$

$$\text{(b)} \quad \rho_{air} = \frac{p_{air}}{R_{air}T} = \frac{101350}{287(293)} \approx 1.205\ \frac{kg}{m^3}.$$

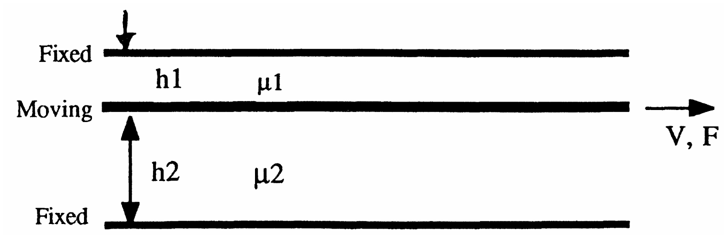
Then the respective gas weights are

$$W_{He} = \rho_{He}g\upsilon = \left(0.1832\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(42412\text{ m}^3) \approx \mathbf{76000\ N} \quad \textit{Ans.}\text{ (a)}$$

$$W_{air} = \rho_{air}g\upsilon = (1.205)(9.81)(42412) \approx \mathbf{501000\ N} \quad \textit{Ans.}\text{ (b)}$$

The difference between these two, **425000 N**, is the *buoyancy*, or lifting ability, of the blimp. [See Section 2.8 for the principles of buoyancy.]

1.48 A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?



Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2}\right) A \quad \textit{Ans.}\ \text{(a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is *no necessary relation* between the two viscosities.

1.71* A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. For an isothermal process, express D_3 as a function of D_1, D_2, p_{atm}, and surface tension Y.

Solution: The masses remain the same for an isothermal process of an ideal gas:

$$m_1 + m_2 = \rho_1 \upsilon_1 + \rho_2 \upsilon_2 = m_3 = \rho_3 \upsilon_3,$$

$$\text{or:} \quad \left(\frac{p_a + 4Y/r_1}{RT}\right)\left(\frac{\pi}{6}D_1^3\right) + \left(\frac{p_a + 4Y/r_2}{RT}\right)\left(\frac{\pi}{6}D_2^3\right) = \left(\frac{p_a + 4Y/r_3}{RT}\right)\left(\frac{\pi}{6}D_3^3\right)$$

The temperature cancels out, and we may clean up and rearrange as follows:

$$\mathbf{p_a D_3^3 + 8YD_3^2 = \left(p_a D_2^3 + 8YD_2^2\right) + \left(p_a D_1^3 + 8YD_1^2\right)} \quad \textit{Ans.}$$

This is a cubic polynomial with a known right hand side, to be solved for D_3.

1.74 Oil, with a vapor pressure of 20 kPa, is delivered through a pipeline by equally-spaced pumps, each of which increases the oil pressure by 1.3 MPa. Friction losses in the pipe are 150 Pa per meter of pipe. What is the maximum possible pump spacing to avoid cavitation of the oil?

Solution: The absolute maximum length L occurs when the pump inlet pressure is slightly greater than 20 kPa. The pump increases this by 1.3 MPa and friction drops the pressure over a distance L until it again reaches 20 kPa. In other words, quite simply,

$$1.3\text{ MPa} = 1{,}300{,}000\text{ Pa} = (150\text{ Pa/m})\text{L}, \quad \text{or} \quad \text{L}_{\text{max}} \approx \mathbf{8660\ m} \quad \textit{Ans.}$$

It makes more sense to have the pump inlet at 1 atm, not 20 kPa, dropping L to about 8 km.

1.78 Sir Isaac Newton measured sound speed by timing the difference between seeing a cannon's puff of smoke and hearing its boom. If the cannon is on a mountain 5.2 miles away, estimate the air temperature in °C if the time difference is (a) 24.2 s; (b) 25.1 s.

Solution: Cannon booms are finite (shock) waves and travel slightly faster than sound waves, but what the heck, assume it's close enough to sound speed:

$$\text{(a)} \quad a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{24.2} = 345.8\ \frac{\text{m}}{\text{s}} = \sqrt{1.4(287)T}, \quad T \approx 298\text{ K} \approx \mathbf{25°C} \quad \textit{Ans.}\text{ (a)}$$

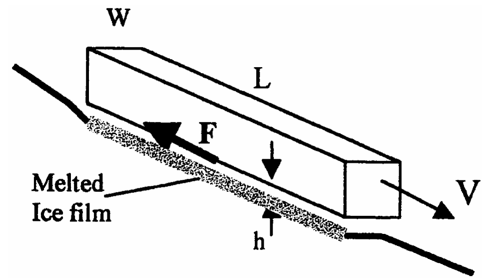
$$\text{(b)} \quad a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{25.1} = 333.4\ \frac{\text{m}}{\text{s}} = \sqrt{1.4(287)T}, \quad T \approx 277\text{ K} \approx \mathbf{4°C} \quad \textit{Ans.}\text{ (b)}$$

C1.2 When a person ice-skates, the ice surface actually melts beneath the blades, so that he or she skates on a thin film of water between the blade and the ice. (a) Find an expression for total friction force F on the bottom of the blade as a function of skater velocity V, blade length L, water film thickness h, water viscosity μ, and blade width W. (b) Suppose a skater of mass *m*, moving at constant speed V_o, suddenly stands stiffly with skates pointed directly forward and allows herself to coast to a stop. Neglecting air resistance, how far will she travel (on *two* blades) before she stops? Give the answer X as a function of (V_o, *m*, L, h, μ, W). (c) Compute X for the case V_o = 4 m/s, m = 100 kg, L = 30 cm, W = 5 mm, and h = 0.1 mm. Do you think our assumption of negligible air resistance was a good one?

Solution: (a) The skate bottom and the melted ice are like two parallel plates:

$$\tau = \mu\frac{V}{h}, \quad F = \tau A = \frac{\mu VLW}{h} \quad \textit{Ans.}\text{ (a)}$$

(b) Use **F** = m**a** to find the stopping distance:



$$\Sigma F_x = -F = -\frac{2\mu VLW}{h} = ma_x = m\frac{dV}{dt}$$

(*the* '2' *is for two blades*)

Separate and integrate once to find the velocity, once again to find the distance traveled:

$$\int \frac{dV}{V} = -\int \frac{2\mu LW}{mh}\,dt, \quad or: \quad V = V_o e^{\frac{-2\mu LW}{mh}t}, \quad X = \int_0^\infty V\,dt = \frac{V_o mh}{2\mu LW} \quad \textit{Ans.} \text{ (b)}$$

(c) Apply our specific numerical values to a 100-kg (!) person:

$$X = \frac{(4.0\ m/s)(100\ kg)(0.0001\ m)}{2(1.788E\text{–}3\ kg/m\cdot s)(0.3\ m)(0.005\ m)} = \mathbf{7460\ m\ (!)} \quad \textit{Ans.} \text{ (c)}$$

We could coast to the next town on ice skates! It appears that our assumption of negligible air drag was grossly incorrect.
