8.29 A uniform water stream, U_∞ = 20 m/s and ρ = 998 kg/m^3, combines with a source at the origin to form a half-body. At (x, y) = (0, 1.2 m), the pressure is 12.5 kPa less than p_∞. (a) Is this point outside the body? Estimate (b) the appropriate source strength *m* and (c) the pressure at the nose of the body.

Solution: We know, from Fig. 8.5 and Eq. 8.18, the point on the half-body surface just above "*m*" is at y = πa/2, as shown, where a = *m*/U. The Bernoulli equation allows us to compute the necessary source strength *m* from the pressure at (x, y) = (0, 1.2 m):

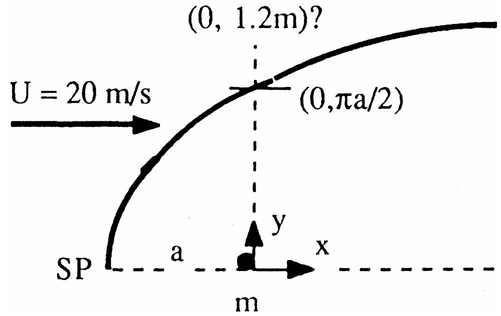

Fig. P8.29

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_\infty + \frac{998}{2}(20)^2 = p_\infty - 12500 + \frac{998}{2}\left[(20)^2 + \left(\frac{m}{1.2}\right)^2\right]$$

Solve for $\boldsymbol{m \approx 6.0\ \frac{m^2}{s}}$ *Ans.* (b) while $a = \frac{m}{U} = \frac{6.0}{20} = 0.3\text{ m}$

The body surface is thus at y = πa/2 = **0.47 m** above m. Thus the point in question, y = 1.2 m above *m*, is **outside the body**. *Ans.* (a)

At the nose SP of the body, (x, y) = (–a, 0), the velocity is zero, hence we predict

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_\infty + \frac{998}{2}(20)^2 = p_{nose} + \frac{\rho}{2}(0)^2, \quad \text{or} \quad \mathbf{p_{nose} \approx p_\infty + 200\ kPa} \quad \textit{Ans. (c)}$$

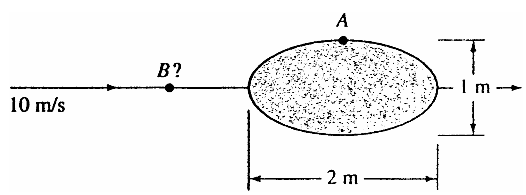

Fig. P8.37

8.37 A Rankine oval 2 m long and 1 m high is immersed in a stream U_∞ = 10 m/s, as in Fig. P8.37. Estimate (a) the velocity at point A and (b) the location of point B where a particle approaching the stagnation point achieves its maximum deceleration.

Solution: (a) With L/h = 2.0, we may evaluate Eq. (8.30) to find the source-sink strength:

$$\frac{h}{a} = \cot\left[\frac{h/a}{2m/(U_\infty a)}\right] \quad \text{and} \quad \frac{L}{a} = \left(1 + \frac{2m}{U_\infty a}\right)^{1/2}$$

$$\text{converges to} \quad \frac{L}{h} = 2.0 \quad \text{if} \quad \frac{m}{U_\infty a} = \mathbf{0.3178}$$

$$\text{Meanwhile,} \quad \frac{h}{a} = 0.6395 \ \text{and} \ \frac{L}{a} = 1.2789 \quad \text{thus} \ a = \frac{1\text{ meter}}{1.2789} \approx \mathbf{0.782\ m}$$

Also compute $V_{max}/U_\infty = 1.451$, hence $V_{max} = 1.451(10) \approx \mathbf{14.5\ m/s}$. *Ans.* (a)

(b) Along the x-axis, at any $x \le -L$, the velocity toward the body nose has the form

$$u = U_\infty + \frac{m}{a+x} + \frac{m}{a-x}, \quad \text{where } m \approx 0.3178 U_\infty a$$

$$\text{Then} \quad \frac{du}{dt} = u\frac{\partial u}{\partial x} = \left[U_\infty + \frac{m}{a+x} + \frac{m}{a-x}\right](-m)\left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right]$$

For this value of m, the maximum deceleration occurs at $\mathbf{x = -1.41a}$ *Ans.*

This is quite near the nose (which is at x = –1.28a). The numerical value of the maximum deceleration is $\mathbf{(du/dt)_{max} \approx -0.655U_\infty^2/a}$.

8.41 A Kelvin oval is formed by a line-vortex pair with K = 9 m^2/s, a = 1 m, and U = 10 m/s. What are the height, width, and shoulder velocity of this oval?

Solution: With reference to Fig. 8.12 and Eq. (8.41), the oval is described by

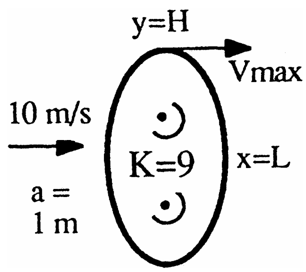

Fig. P8.41

$$\psi = 0,\ x = 0,\ y = H: \quad UH = \frac{K}{2}\ln\left[\frac{(H+a)^2}{(H-a)^2}\right], \quad \text{with } \frac{K}{Ua} = \frac{9}{10(1)} = \mathbf{0.9}$$

Solve by iteration for $H/a \approx 1.48$, or $\mathbf{2H}$ = oval height $\approx$ **2.96 m** *Ans.*

$$\text{Similarly,} \quad \frac{L}{a} = \left(\frac{2K}{Ua} - 1\right)^{1/2} = [2(0.9)-1]^{1/2} = 0.894, \quad \mathbf{2L} = \text{width} \approx \mathbf{1.79\ m} \quad \textit{Ans.}$$

$$\text{Finally,} \quad V_{max} = U + \frac{K}{H-a} - \frac{K}{H+a} = 10 + \frac{9}{0.48} - \frac{9}{2.48} \approx \mathbf{25.1\ \frac{m}{s}} \quad \textit{Ans.}$$

8.50 It is desired to simulate flow past a ridge or "bump" by using a streamline *above* the flow over a cylinder, as shown in Fig. P8.50. The bump is to be $a/2$ high, as shown. What is the proper elevation h of this streamline? What is U_{max} on the bump compared to U_∞?

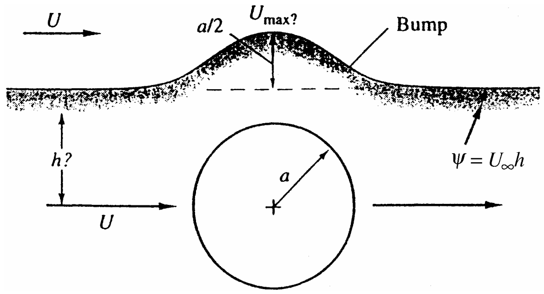

Fig. P8.50

Solution: Apply the equation of the streamline (Eq. 8.32) to θ=180° and also 90°:

$$\psi = U_\infty \sin\theta\left(r - \frac{a^2}{r}\right) \quad \text{at } \theta = 180° \text{ (the freestream) gives } \psi = U_\infty h$$

$$\text{Then, at } \theta = 90°, \quad r = h + \frac{a}{2}, \quad \psi = U_\infty h = U_\infty \sin 90°\left(h + \frac{a}{2} - \frac{a^2}{h + a/2}\right)$$

$$\text{Solve for } \quad \mathbf{h = \frac{3}{2}a} \quad \textit{Ans.} \quad \text{(corresponds to } r = 2a)$$

The velocity at the hump ($r = 2a$, $\theta = 90°$) then follows from Eq. (8.33):

$$U_{max} = U_\infty \sin 90°\left[1 + \frac{a^2}{(2a)^2}\right] \quad \text{or} \quad \mathbf{U_{max} = \frac{5}{4}U_\infty} \quad \textit{Ans.}$$

8.75 Using the four-source image pattern needed to construct the flow near a corner shown in Fig. P8.72, find the value of the source strength m which will induce a wall velocity of 4.0 m/s at the point $(x, y) = (a, 0)$ just below the source shown, if a = 50 cm.

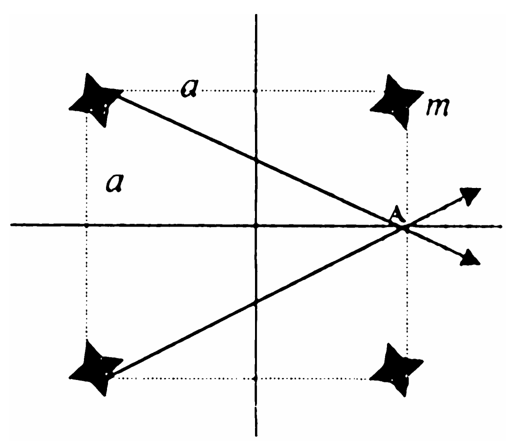

Fig. P8.75

Solution: The flow pattern is formed by four equal sources m in the 4 quadrants, as in the figure at right. The sources above and below the point A(a, 0) cancel each other at A, so the velocity at A is caused only by the two left sources. The velocity at A is the sum of the two horizontal components from these 2 sources:

$$V_A = 2\frac{m}{\sqrt{a^2 + (2a)^2}}\frac{2a}{\sqrt{a^2 + (2a)^2}} = \frac{4ma}{5a^2} = \frac{4m}{5(0.5m)} = 4\ \frac{\text{m}}{\text{s}} \quad \text{if} \quad \boldsymbol{m} = \mathbf{2.5\ \frac{m^2}{s}} \qquad \textit{Ans.}$$
