P7.9 Repeat the flat-plate momentum analysis of Sec. 7.2 by replacing the parabolic profile, Eq. (7.6), with the more accurate sinusoidal profile:

$$\frac{u}{U} \approx \sin\left(\frac{\pi y}{2\delta}\right)$$

Compute momentum-integral estimates of C_f, δ/x, δ^*/x, and H.

Solution: Carry out the same integrations as Section 7.2, but results are more accurate:

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy \approx \frac{4-\pi}{2\pi}\delta = 0.1366\delta; \quad \delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy \approx \frac{\pi-2}{\pi}\delta = 0.3634\delta$$

$$\tau_w \approx \mu\frac{\pi U}{2\delta} = \rho U^2 \frac{d}{dx}\left[\frac{4-\pi}{2\pi}\delta\right], \quad \text{integrate to:} \quad \frac{\delta}{x} \approx \frac{\pi\sqrt{2}/\sqrt{(4-\pi)}}{\sqrt{Re_x}} \approx \frac{4.80}{\sqrt{Re_x}} \quad (5\% \text{ low})$$

Substitute these results back to obtain the desired (accurate) dimensionless expressions:

$$\frac{\delta}{x} \approx \frac{\mathbf{4.80}}{\sqrt{\mathbf{Re_x}}}; \quad C_f = \frac{\theta}{x} \approx \frac{\mathbf{0.655}}{\sqrt{\mathbf{Re_x}}}; \quad \frac{\delta^*}{x} \approx \frac{\mathbf{1.743}}{\sqrt{\mathbf{Re_x}}}; \quad H = \frac{\delta^*}{\theta} \approx \mathbf{2.66} \quad \textit{Ans.} \text{ (a, b, c, d)}$$

P7.20 Air at 20°C and 1 atm flows at 20 m/s past the flat plate in Fig. P7.20. A pitot stagnation tube, placed 2 mm from the wall, develops a manometer head *h* = 16 mm of Meriam red oil, SG = 0.827. Use this information to estimate the downstream position x of the pitot tube. Assume laminar flow.

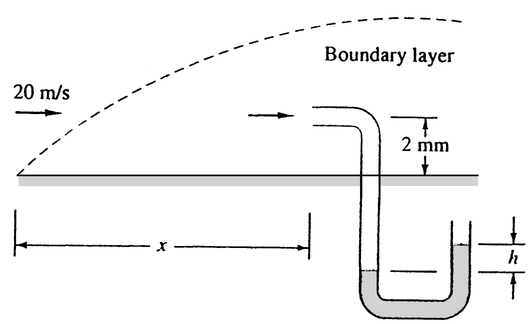

Fig. P7.20

Solution: For air at 20°C, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. Assume constant stream pressure, then the manometer can be used to estimate the local velocity u at the position of the pitot inlet:

$$\Delta p_{mano} = p_o - p_\infty = (\rho_{oil} - \rho_{air})gh_{mano} = [0.827(998) - 1.2](9.81)(0.016) \approx 129 \text{ Pa}$$

$$\text{Then} \quad u_{pitot\ inlet} \approx [2\Delta p/\rho]^{1/2} = [2(129)/1.2]^{1/2} \approx 14.7 \text{ m/s}$$

Now, with u known, the Blasius solution uses u/U to determine the position η:

$$\frac{u}{U} = \frac{14.7}{20} = 0.734, \quad \text{Table 7.1 read } \eta \approx 2.42 = y(U/\nu x)^{1/2}$$

$$\text{or:} \quad x = (U/\nu)(y/\eta)^2 = (20/1.5E{-}5)(0.002/2.42)^2 \approx \mathbf{0.908\ m} \quad \textit{Ans.}$$

Check Re_x = (20)(0.908)/(1.5E–5) ≈ 1.21E6, OK, laminar if the flow is very smooth.

P7.45 A thin sheet of fiberboard weighs 90 N and lies on a rooftop, as shown in the figure. Assume ambient air at 20°C and 1 atm. If the coefficient of solid friction between board and roof is $\sigma = 0.12$, what wind velocity will generate enough friction to dislodge the board?

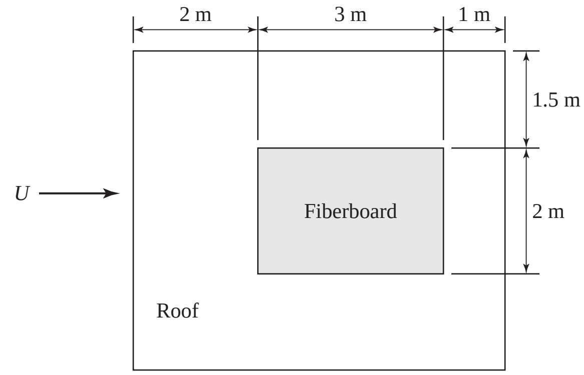

Fig. P7.45

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-5}$ kg/m·s. Our first problem is to evaluate the drag when the leading edge is *not* at x = 0. Since the dimensions are large, we will assume that the flow is *turbulent* and check this later:

$$F = \int_{x_1}^{x_2} \tau_w \, dA = \int_{x_1}^{x_2} \left[\frac{0.027(\rho/2)U^2}{(\rho U x/\mu)^{1/7}} \right] b\,dx = \left(\frac{0.031 b \rho U^2}{2} \right) \left(\frac{\mu}{\rho U} \right)^{1/7} \left(x_2^{6/7} - x_1^{6/7} \right)$$

Set this equal to the dislodging friction force $F = \sigma W = 0.12(90) = 10.8$ N:

$$\frac{0.031}{2}(1.2)(2.0)U^2 \left(\frac{1.8E{-5}}{1.2U} \right)^{1/7} (5.0^{6/7} - 2.0^{6/7}) = 10.8 \text{ N}$$

Solve this for U = **33 m/s ≈ 73 mi/h** *Ans.*

$\text{Re}_{x_1} = 4.4\text{E}6$: turbulent, OK.

P7.75 The helium-filled balloon in Fig. P7.75 is tethered at 20°C and 1 atm with a string of negligible weight and drag. The diameter is 50 cm, and the balloon material weighs 0.2 N, not including the helium. The helium pressure is 120 kPa. Estimate the tilt angle θ if the airstream velocity U is (a) 5 m/s or (b) 20 m/s.

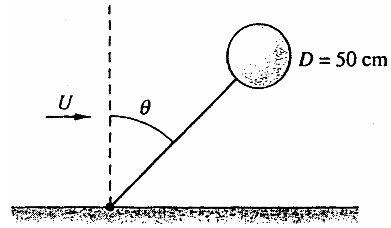

Fig. P7.75

Solution: For air at 20°C and 1 atm, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. For helium, R = 2077 J/kg·°K. The helium density = (120000)/[2077(293)] ≈ 0.197 kg/m^3.

The balloon net buoyancy is independent of the flow velocity:

$$B_{net} = (\rho_{air} - \rho_{He})g\frac{\pi}{6}D^3 = (1.2 - 0.197)(9.81)\frac{\pi}{6}(0.5)^3 \approx 0.644 \text{ N}$$

The net upward force is thus $F_z = (B_{net} - W) = 0.644 - 0.2 = 0.444$ N. The balloon drag *does* depend upon velocity. At 5 m/s, we expect laminar flow:

$$\text{(a) } U = 5\ \frac{m}{s}:\ \ \text{Re}_D = \frac{1.2(5)(0.5)}{1.8E{-}5} = 167000; \quad \textit{Table } 7.3:\ C_D \approx 0.47$$

$$Drag = C_D\frac{\rho}{2}U^2\frac{\pi}{4}D^2 = 0.47\left(\frac{1.2}{2}\right)(5)^2\frac{\pi}{4}(0.5)^2 \approx 1.384 \text{ N}$$

$$\text{Then} \quad \theta_a = \tan^{-1}\left(\frac{Drag}{F_z}\right) = \tan^{-1}\left(\frac{1.384}{0.444}\right) = \mathbf{72°} \quad \textit{Ans. (a)}$$

(b) At 20 m/s, Re = 667000 (*turbulent*), Table 7.3: $C_D \approx 0.2$:

$$Drag = 0.2\left(\frac{1.2}{2}\right)(20)^2\frac{\pi}{4}(0.5)^2 = 9.43 \text{ N}, \quad \theta_b = \tan^{-1}\left(\frac{9.43}{0.444}\right) = \mathbf{87°} \quad \textit{Ans. (b)}$$

These angles are too steep—the balloon needs more buoyancy and/or less drag.