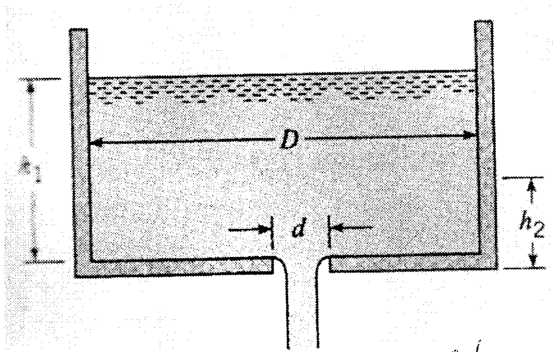
It takes a certain length of time for the liquid level in a tank of diameter D to drop from position h_1 to position h_2 as the tank is being drained through an orifice of diameter d at the bottom. Determine the pi groups that apply to this problem. Assume that the liquid is nonviscous. Express your answer in the function form:

$$\frac{\Delta h}{d} = f\left(\pi_1, \pi_2, \pi_3\right)$$



$$\frac{\Delta h}{d} = f\left(\frac{D}{d}, \frac{\gamma t^2}{\rho d}, \frac{h}{d}\right)$$

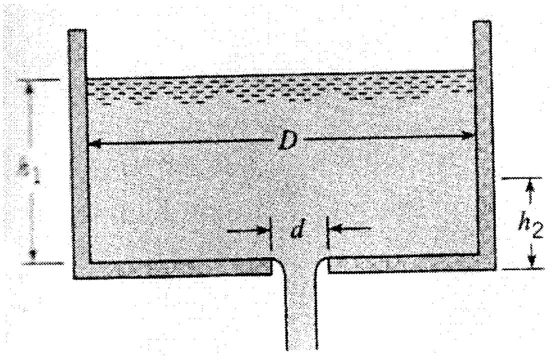
take out L — take out T

Δh	L	$\Delta h/d$	0	$\Delta h/d$	0	$\Delta h/d$	0
t	T	t	T	t	T		
ρ	M/L^3	ρd^3	M				
D	L	D/d	0	D/d	0	D/d	0
d	L						
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
h	L	h/d	0	h/d	0	h/d	0

take out M

It takes a certain length of time for the liquid level in a tank of diameter D to drop from position h_1 to position h_2 as the tank is being drained through an orifice of diameter d at the bottom. Determine the pi groups that apply to this problem. Assume that the liquid is nonviscous. Express your answer in the function form:

$$\frac{\Delta h}{d} = f\left(\pi_1, \pi_2, \pi_3\right)$$



solution: exponent method $F(\Delta h, t, \rho, D, d, \gamma, h)$ $n=7$

$\Delta h: L$, $t: T$, $\rho: \frac{M}{L^3}$, $D: L$, $d: L$, $\gamma: \frac{M}{L^2T^2}$, $h: L$

repeat variable ρ, d, t ∴ $n-m=7-3=4$

$\pi_1 = \rho^{x_1} d^{y_1} t^{z_1} \Delta h = \left(\frac{M}{L^3}\right)^{x_1}(L)^{y_1}(T)^{z_1} L$ $\quad -3x_1+y_1+1=0$ $\quad x_1=0,\ z_1=0$ ∴ $y_1=-1$

$\boxed{\pi_1 = \Delta h/d}$

$\pi_2 = \rho^{x_2} d^{y_2} t^{z_2} D = \left(\frac{M}{L^3}\right)^{x_2}(L)^{y_2}(T)^{z_2} L$ $\quad x_2=z_2=0,\ y_2=-1$

$\boxed{\pi_2 = D/d}$

$\pi_3 = \rho^{x_3} d^{y_3} t^{z_3} \gamma = \left(\frac{M}{L^3}\right)^{x_3}(L)^{y_3}(T)^{z_3}\left(\frac{M}{L^2T^2}\right)$ $\quad x_3+1=0,\ -3x_3+y_3-2=0,\ z_3-2=0$ $\quad x_3=-1,\ y_3=-1,\ z_3=2$

$\boxed{\pi_3 = \frac{\gamma t^2}{\rho d}}$

$\pi_4 = \rho^{x_4} d^{y_4} t^{z_4} h = \left(\frac{M}{L^3}\right)^{x_4} L^{y_4} T^{z_4} L$ $\quad x_4=0,\ z_4=0,\ y_4=-1$

$= \cancel{d}(h/d)$

8.3 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

Δh	L	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0
t	T	t	T	t	T		
ρ	$\frac{M}{L^3}$	ρd^3	M				
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
h	L	$\frac{h}{d}$	0	$\frac{h}{d}$	0	$\frac{h}{d}$	0

In the first step, length is taken out with d. In the second step, mass is taken out with ρd^3. In the third step, time is taken out with t. The functional relationship is

$$\frac{\Delta h}{d} = f(\frac{D}{d}, \frac{\gamma t^2}{\rho d}, \frac{h}{d})$$

This can also be written as

$$\frac{\Delta h}{d} = f(\frac{d}{D}, \frac{gt^2}{d}, \frac{h}{d})$$