5.15) $\tau_w = f(U, \delta, u', \rho, \frac{dP}{dx})$ rewrite this relationship as a dimensionless function.

$\{\tau_w\} = \{ML^{-1}T^{-2}\}$ Table 5.1

$\{U\} = \{LT^{-1}\}$

$\{\delta\} = \{L\}$

$\{u'\} = \{LT^{-1}\}$

$\{\rho\} = \{ML^{-3}\}$

$\{\frac{dP}{dx}\} = \{\frac{ML^{-1}T^{-2}}{L}\} = \{ML^{-2}T^{-2}\}$

$n = 6$
$j = 3 \Rightarrow k = 6 - 3 = 3$

Hint: repeating variables are better to be U, ρ, δ

$$\Pi_1 = \rho^{a_1} U^{b_1} \delta^{c_1} \tau_w = \left\{\frac{M}{L^3}\right\}^{a_1} \left\{\frac{L}{T}\right\}^{b_1} \{L\}^{c_1} \left\{\frac{M}{LT^2}\right\} = M^0 L^0 T^0$$

$\overset{M}{\Rightarrow} \quad a_1 + 1 = 0 \Rightarrow a_1 = -1$

$\overset{T}{\Rightarrow} \quad -b_1 - 2 = 0 \Rightarrow b_1 = -2$

$\overset{L}{\Rightarrow} \quad -3a_1 + b_1 + c_1 - 1 = 0 \Rightarrow 3 - 2 + c_1 - 1 = 0 \Rightarrow c_1 = 0$

$$\Rightarrow \Pi_1 = \frac{\tau_w}{\rho U^2}$$

$$\Pi_2 = \rho^{a_2} U^{b_2} \delta^{c_2} u' = \left\{\frac{M}{L^3}\right\}^{a_2} \left\{\frac{L}{T}\right\}^{b_2} \{L\}^{c_2} \left\{\frac{L}{T}\right\} = M^0 L^0 T^0$$

M: $a_2 + 0 = 0 \Rightarrow a_2 = 0$

T: $-b_2 + 1 = 0 \Rightarrow b_2 = -1$

L: $b_2 + c_2 + 1 = 0 \Rightarrow c_2 = -b_2 - 1 = 0$

$$\Rightarrow \Pi_2 = \frac{u'}{U}$$

$$\Pi_3 = \rho^{a_3} U^{b_3} \delta^{c_3} \frac{dP}{dx} = \left\{\frac{M}{L^3}\right\}^{a_3} \left\{\frac{L}{T}\right\}^{b_3} \{L\}^{c_3} \left\{\frac{M}{L^2T^2}\right\} = M^0 L^0 T^0$$

M: $a_3 + 1 = 0 \Rightarrow a_3 = -1$

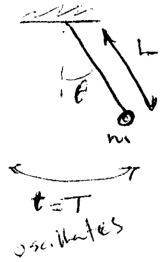
T: $-b_3 - 2 = 0 \Rightarrow b_3 = -2$

L: $-3a_3 + b_3 + c_3 - 2 = 0 \Rightarrow 3 - 2 + c_3 - 2 = 0$
$\Rightarrow c_3 = 1$

$$\Rightarrow \Pi_3 = \frac{dP}{dx} \frac{\delta}{\rho U^2}$$

$$\Rightarrow \Pi_1 = fn(\Pi_2, \Pi_3) \Rightarrow \frac{\tau_w}{\rho U^2} = f\left(\frac{u'}{U}, \frac{dP}{dx}\frac{\delta}{\rho U^2}\right)$$

5.26)



on earth: $L = 1\,m$, $m = 200\,g$, $\theta = 20°$, $g = 9.81$ $\rightarrow T = 2.04\,s$

a) what is period if $\theta = 45°$?

b) on moon: $m = 100\,g$, $L = 30\,cm$, $\theta = 20°$, $g = 1.62\,m/s^2$ $\rightarrow T = ?$

$$T = f(L, m, g, \theta)$$

$$\{T\} \quad \{L\} \quad \{M\} \quad \{LT^{-2}\} \quad \{1\}$$

$n = 5$, $j = 3$ $\Rightarrow k = 2$

$\pi_1 = \theta$ θ is dimensionless

$$\pi_2 = T L^{a_2} g^{b_2} M^{c_2} = \{T\}\{L\}^{a_2}\left\{\frac{L}{T^2}\right\}^{b_2}\{M\}^{c_2} = \{M^0 L^0 T^0\}$$

$\Rightarrow$ M: $c_2 = 0$

T: $1 - 2b_2 = 0 \Rightarrow b_2 = \frac{1}{2}$ $\Rightarrow \pi_2 = T\sqrt{\frac{g}{L}}$

L: $a_2 + b_2 = 0 \Rightarrow a_2 = -\frac{1}{2}$

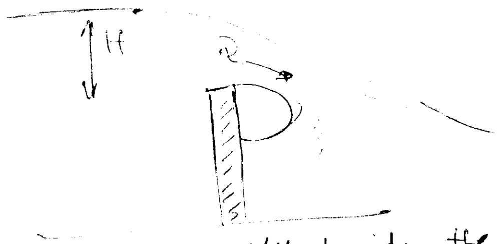
$$\pi_2 = f(\pi_1) \Rightarrow T\sqrt{\frac{g}{L}} = f(\theta)$$

a) if we change θ, we cannot find T since we don't know about f.

b) At same θ: $T_1\sqrt{\frac{g_1}{L_1}} = T_2\sqrt{\frac{g_2}{L_2}}$

$$\Rightarrow 2.04\sqrt{\frac{9.81}{1}} = T_2\sqrt{\frac{1.62}{0.3}} \Rightarrow \underline{T_2 = 2.75\,s}$$

5.32)



width b into the board

$Q = f(g, b, H)$

Also, $Q \propto b$ $\Rightarrow$ find a nondimensional relationship

$$Q = f(g, b, H) \qquad n = 4,\ j = 2 \Rightarrow k = 4 - 2 = 2$$

$$\{L^3T^{-1}\} \quad \{LT^{-2}\}\ \{L\}\ \{L\}$$

repeating variables: g & H

$$\pi_1 = Q g^{a_1} H^{b_1} = \{L^3T^{-1}\}\{LT^{-2}\}^{a_1}\{L\}^{b_1}$$

T: $-1 - 2a_1 = 0 \Rightarrow a_1 = -\frac{1}{2}$

L: $3 + a_1 + b_1 = 0 \Rightarrow b_1 = -\frac{5}{2}$

$$\Rightarrow \pi_1 = \frac{Q}{g^{1/2} H^{5/2}}$$

$$\pi_2 = b\, g^{a_2} H^{b_2} \Rightarrow \{L\}\{LT^{-2}\}^{a_2}\{L\}^{b_2}$$

T: $-2a_2 = 0 \Rightarrow a_2 = 0$

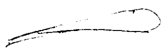
L: $1 + a_2 + b_2 = 0 \Rightarrow b_2 = -1$

$$\Rightarrow \pi_2 = \frac{b}{H}$$

$$\pi_1 = f(\pi_2) \Longrightarrow \frac{Q}{g^{1/2} H^{5/2}} = f\left(\frac{b}{H}\right)$$

if $Q \propto b \Rightarrow f\left(\frac{b}{H}\right) = \text{const.}\ \frac{b}{H} \Longrightarrow \frac{Q}{g^{1/2} H^{3/2}} = c.\ \frac{b}{H}$

$$\frac{Q}{b g^{1/2} H^{3/2}} = \text{Const.}$$

5-74)  wing

$L_m = \frac{1}{10} L_p$ $V_m = 700$ m/s at $P = 1$ atm, pitching moment $M_m = 0.25$ kN·m

if Re number effects are negligible, $M_p = ?$ at same Mach number at 8 km altitude?

$M = f(V, l, \rho, \mu, a, \alpha) = f(Re, Ma, \alpha)$

$Ma = \frac{V}{a}$ For sea-level air $\rho = 1.225$ kg/m^3 and $a = 340$ Table A-6.

For $h = 8000$ m $\rho = 0.525$ " " $a = 308$

$Ma_m = Ma_p \Rightarrow \frac{V_m}{a_m} = \frac{V_p}{a_p} \Rightarrow \frac{700}{340} = \frac{V_p}{308} \Rightarrow V_p = 634$ m/s

$M = \{ML^2T^{-2}\}$ $C_D = \frac{F}{\frac{1}{2}\rho L^2 V^2}$ $C_M = \frac{F \cdot L}{\frac{1}{2}\rho L^3 V^2} = \frac{M}{\frac{1}{2}\rho L^3 V^2}$

~~$C_M = C_D L$~~

$$\frac{M_m}{\frac{1}{2}\rho_m L_m^3 V_m^2} = \frac{M_p}{\frac{1}{2}\rho_p L_p^3 V_p^2} \Rightarrow M_p = M_m \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^3$$

$$= 0.25 \times \frac{0.525}{1.225} \times \left(\frac{634}{700}\right)^2 \times 10^3$$

$$= 88 \text{ kN·m}$$