3.16 An incompressible fluid flows past an impermeable flat plate, as in Fig. P3.16, with a uniform inlet profile $u = U_o$ and a cubic polynomial exit profile

$$u \approx U_o\left(\frac{3\eta - \eta^3}{2}\right) \quad \text{where } \eta = \frac{y}{\delta}$$

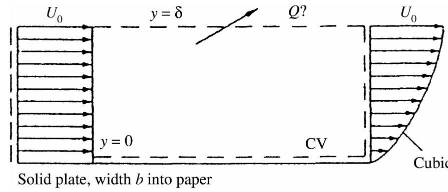

Fig. P3.16

Compute the volume flow Q across the top surface of the control volume.

Solution: For the given control volume and incompressible flow, we obtain

$$0 = Q_{top} + Q_{right} - Q_{left} = Q + \int_0^{\delta} U_o\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right) b\,dy - \int_0^{\delta} U_o b\,dy$$

$$= Q + \frac{5}{8}U_o b\delta - U_o b\delta, \quad \text{solve for } \mathbf{Q = \frac{3}{8}U_o b\delta} \quad \textit{Ans.}$$

3.28 According to Torricelli's theorem, the velocity of a fluid draining from a hole in a tank is $V \approx (2gh)^{1/2}$, where h is the depth of water above the hole, as in Fig. P3.28. Let the hole have area A_o and the cylindrical tank have bottom area A_b. Derive a formula for the time to drain the tank from an initial depth h_o.

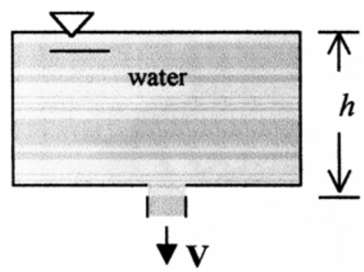

Fig. P3.28

Solution: For a control volume around the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} = 0$$

$$\rho A_b \frac{dh}{dt} = -\dot{m}_{out} = -\rho A_o\sqrt{2gh}$$

$$\int_{h_o}^{0} \frac{dh}{\sqrt{h}} = \int_0^t \frac{A_o\sqrt{2g}}{A_b}\,dt; \qquad \boldsymbol{t = \frac{A_b}{A_o}\sqrt{\frac{h_o}{2g}}} \quad \textit{Ans.}$$

3.54 For the pipe-flow reducing section of Fig. P3.54, D_1 = 8 cm, D_2 = 5 cm, and p_2 = 1 atm. All fluids are at 20°C. If V_1 = 5 m/s and the manometer reading is *h* = 58 cm, estimate the total horizontal force resisted by the flange bolts.

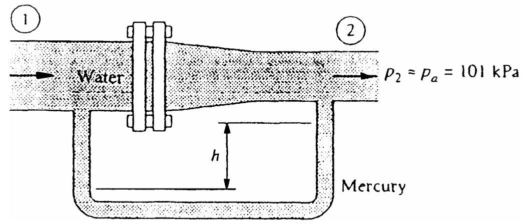

Fig. P3.54

Solution: Let the CV cut through the bolts and through section 2. For the given manometer reading, we may compute the upstream pressure:

$$p_1 - p_2 = (\gamma_{merc} - \gamma_{water})h = (132800 - 9790)(0.58\text{ m}) \approx 71300\text{ Pa (gage)}$$

Now apply conservation of mass to determine the exit velocity:

$$Q_1 = Q_2, \quad \text{or} \quad (5\text{ m/s})(\pi/4)(0.08\text{ m})^2 = V_2(\pi/4)(0.05)^2, \quad \text{solve for } V_2 \approx 12.8\text{ m/s}$$

Finally, write the balance of horizontal forces:

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or:} \quad F_{bolts} = (71300)\frac{\pi}{4}(0.08)^2 - (998)\frac{\pi}{4}(0.08)^2(5.0)[12.8 - 5.0] \approx \mathbf{163\ N} \quad \textit{Ans.}$$

3.55 In Fig. P3.55 the jet strikes a vane which moves to the right at constant velocity V_c on a frictionless cart. Compute (a) the force F_x required to restrain the cart and (b) the power *P* delivered to the cart. Also find the cart velocity for which (c) the force F_x is a maximum and (d) the power *P* is a maximum.

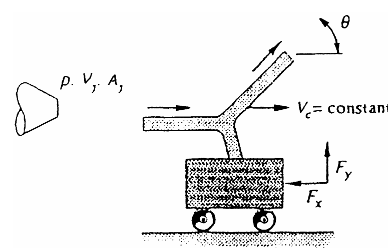

Fig. P3.55

Solution: Let the CV surround the vane and cart and move to the right at cart speed. The jet strikes the vane at *relative* speed $V_j - V_c$. The cart does not accelerate, so the horizontal force balance is

$$\sum F_x = -F_x = [\rho A_j(V_j - V_c)](V_j - V_c)\cos\theta - \rho A_j(V_j - V_c)^2$$

$$\text{or:} \quad F_x = \boldsymbol{\rho A_j(V_j - V_c)^2(1 - \cos\theta)} \quad \textit{Ans.}\text{ (a)}$$

The power delivered is $P = V_cF_x = \boldsymbol{\rho A_j V_c(V_j - V_c)^2(1 - \cos\theta)}$ *Ans.* (b)

The maximum <u>force</u> occurs when the cart is fixed, or: $V_c = \mathbf{0}$ *Ans.* (c)

The maximum <u>power</u> occurs when $dP/dV_c = 0$, or: $V_c = \mathbf{V_j/3}$ *Ans.* (d)

3.170 If losses are neglected in Fig. P3.170, for what water level *h* will the flow begin to form vapor cavities at the throat of the nozzle?

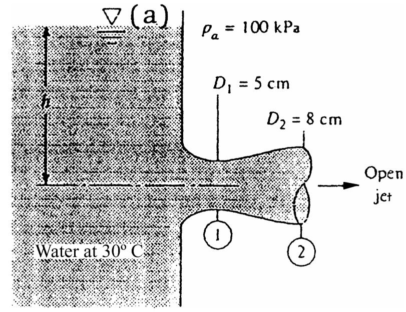

Fig. P3.170

Solution: Applying Bernoulli from (a) to (2) gives Torricelli's relation: $V_2 = \sqrt{2gh}$. Also,

$$V_1 = V_2(D_2/D_1)^2 = V_2(8/5)^2 = 2.56V_2$$

Vapor bubbles form when p_1 reaches the <u>vapor pressure</u> at 30°C, $p_{vap} \approx 4242$ Pa (from Table A.5), while $\rho \approx 996\ kg/m^3$ at 30°C (Table A.1). Apply Bernoulli between 1 and 2:

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1 \approx \frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2, \quad \text{or:} \quad \frac{4242}{996}+\frac{(2.56V_2)^2}{2}+0 \approx \frac{100000}{996}+\frac{V_2^2}{2}+0$$

$$\text{Solve for} \quad V_2^2 = 34.62 = 2gh, \quad \text{or} \quad h = 34.62/[2(9.81)] \approx \mathbf{1.76\ m} \quad \textit{Ans.}$$

3.176 In the spillway flow of Fig. P3.176, the flow is assumed uniform and hydrostatic at sections 1 and 2. If losses are neglected, compute (a) V_2 and (b) the force per unit width of the water on the spillway.

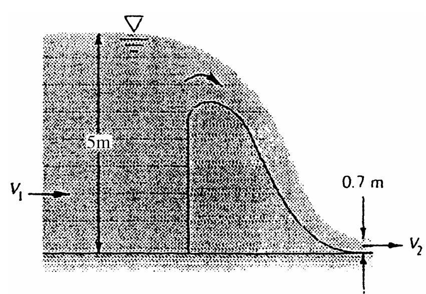

Fig. P3.176

Solution: For mass conservation,

$$V_2 = V_1h_1/h_2 = \frac{5.0}{0.7}V_1 = 7.14V_1$$

(a) Now apply Bernoulli from 1 to 2:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+h_1 \approx \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+h_2; \quad \text{or:} \quad 0+\frac{V_1^2}{2g}+5.0 \approx 0+\frac{(7.14V_1)^2}{2g}+0.7$$

$$\text{Solve for } V_1^2 = \frac{2(9.81)(5.0-0.7)}{[(7.14)^2-1]}, \text{ or } V_1 = \mathbf{1.30\ \frac{m}{s}}, \quad V_2 = 7.14V_1 = \mathbf{9.28\ \frac{m}{s}} \quad \textit{Ans. (a)}$$

(b) To find the force on the spillway (F ←), put a CV around sections 1 and 2 to obtain

$$\sum F_x = -F + \frac{\gamma}{2}h_1^2 - \frac{\gamma}{2}h_2^2 = \dot{m}(V_2 - V_1), \quad \text{or, using the given data,}$$

$$F = \frac{1}{2}(9790)[(5.0)^2-(0.7)^2] - 998[(1.30)(5.0)](9.28-1.30) \approx \mathbf{68300\ \frac{N}{m}} \quad \textit{Ans. (b)}$$

3.178 For the water channel flow of Fig. P3.178, h_1 = 0.45 ft, H = 2.2 ft, and V_1 = 16 ft/s. Neglecting losses and assuming uniform flow at sections 1 and 2, find the downstream depth h_2. Show that *two* realistic solutions are possible.

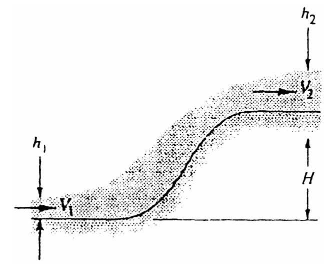

Fig. P3.178

Solution: The analysis is quite similar to Prob. 3.177 - continuity + Bernoulli:

$$V_2 = V_1\frac{h_1}{h_2} = \frac{16(0.45)}{h_2}; \quad \frac{V_1^2}{2g} + h_1 = \frac{V_2^2}{2g} + h_2 + H = \frac{V_1^2}{2(32.2)} + 0.45 = \frac{(7.2/h_2)^2}{2(32.2)} + h_2 + 2.2$$

Combine into a cubic equation: $h_2^3 - 2.225\, h_2^2 + 0.805 = 0$. The three roots are:

$$h_2 = -0.540 \text{ ft (impossible)}; \quad h_2 = +\mathbf{2.03\ ft}\ (\underline{\text{subcritical}});$$
$$h_2 = +\mathbf{0.735\ ft}\ (\underline{\text{supercritical}}) \quad \textit{Ans.}$$