| ID:----------------- | Midterm 2 | Time: 50 minutes |
|---|---|---|
| Name: ------------------ | Course: 58:160, Fall 2009 | |

Solution:
1.

Assumptions: (2)

i. Steady
ii. $u_r$ and $u_\theta$=0
iii. fully developed
iv. pressure gradient is: $-\frac{\Delta p}{l}$
v. no gravity in z direction
vi. axisymmetric

z-momentum:

$$\rho\left(\underbrace{\underbrace{\frac{\partial u_z}{\partial t}}_{=0;assumption1}}_{0.25}+\underbrace{\underbrace{u_r\frac{\partial u_z}{\partial r}}_{=0;assumption2}}_{0.25}+\underbrace{\underbrace{\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}}_{=0;assumption2}}_{0.25}+\underbrace{\underbrace{u_z\frac{\partial u_z}{\partial z}}_{=0;assumption3}}_{0.25}\right)=\underbrace{\underbrace{-\frac{\partial p_z}{\partial z}}_{=\frac{\Delta p}{l};assumption4}}_{0.25}+\underbrace{\underbrace{\rho g_z}_{=0;assumption5}}_{0.25}+\mu\left(\underbrace{\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)}_{unknown}}_{0.25}+\underbrace{\underbrace{\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}}_{=0;assumption6}}_{0.25}+\underbrace{\underbrace{\frac{\partial^2 u_z}{\partial z^2}}_{=0;assumption3}}_{0.25}\right)$$

Therefore:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)=\frac{-\Delta p}{l\mu}\Rightarrow r\frac{\partial u_z}{\partial r}=\frac{-\Delta p}{l\mu}\frac{r^2}{2}+C_1\Rightarrow\frac{\partial u_z}{\partial r}=\frac{-\Delta p}{l\mu}\frac{r}{2}+\frac{C_1}{r}\Rightarrow u_z=\underbrace{\frac{-\Delta p}{l\mu}\frac{r^2}{4}}_{0.25}+\underbrace{C_1\ln r}_{0.25}+\underbrace{C_2}_{0.25}$$

Boundary conditions:

$$\begin{cases}\underbrace{r=r_i: u_z=V_0}_{1}\\ \underbrace{r=r_o: u_z=0}_{1}\end{cases}\Rightarrow\begin{cases}\frac{-\Delta p}{l\mu}\frac{r_i^2}{4}+C_1\ln r_i+C_2=V_0\\ \frac{-\Delta p}{l\mu}\frac{r_o^2}{4}+C_1\ln r_o+C_2=0\end{cases}\Rightarrow C_1=\underbrace{\frac{V_0+\frac{\Delta p}{4l\mu}\left(r_i^2-r_o^2\right)}{\ln\left(r_i/r_o\right)}}_{0.5};\ C_2=\underbrace{\frac{\Delta p}{l\mu}\frac{r_o^2}{4}-\frac{V_0+\frac{\Delta p}{4l\mu}\left(r_i^2-r_o^2\right)}{\ln\left(r_i/r_o\right)}\ln r_o}_{0.5}$$

$$D=\tau_{rz}A\quad D=0\Rightarrow\underbrace{\tau_{rz}(r=r_i)=0}_{0.5}\Rightarrow$$

$$\tau_{rz}=\underbrace{\mu\left(\frac{\partial u_z}{\partial r}+\underbrace{\frac{\partial u_r}{\partial z}}_{=0}\right)_{r=r_i}}_{0.5}=\mu\left(\frac{-\Delta p}{l\mu}\frac{r_i}{2}+\frac{C_1}{r_i}\right)=0\Rightarrow\mu\left(\frac{-\Delta p}{l\mu}\frac{r_i}{2}+\frac{\frac{V_0+\frac{\Delta p}{4l\mu}\left(r_i^2-r_o^2\right)}{\ln\left(r_i/r_o\right)}}{r_i}\right)=0$$

$$\Rightarrow V_0=\frac{\Delta p}{2l\mu}r_i^2\ln\left(r_i/r_o\right)-\frac{\Delta p}{4l\mu}\left(r_i^2-r_o^2\right)=\underbrace{\frac{\Delta p}{4l\mu}\left(2r_i^2\ln\left(r_i/r_o\right)+r_o^2-r_i^2\right)}_{1}$$

(2)

$$\left.\begin{array}{l} Fr_m = Fr_p \Rightarrow \underbrace{\dfrac{V_m}{\sqrt{d_m g_m}} = \dfrac{V_p}{\sqrt{d_p g_p}}}_{2} \\ \underbrace{g_m = g_p}_{1} \\ d_m / d_p = 1/4 \end{array}\right\} \dfrac{V_m}{V_p} = \sqrt{\dfrac{d_m}{d_p}} = \underbrace{\dfrac{1}{2}}_{1}$$

$$\mathrm{Re}_m = \mathrm{Re}_p \Rightarrow \underbrace{\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho_p V_p d_p}{\mu_p}}_{2} \Rightarrow \frac{\mu_m}{\mu_p} = \frac{V_m}{V_p} \times \frac{d_m}{d_p} \times \frac{\rho_m}{\rho_p} = \frac{1}{2} \times \frac{1}{4} \times 2 = \underbrace{\frac{1}{4}}_{1}$$

$$We_m = We_p \Rightarrow \underbrace{\frac{\rho_m V_m^2 d_m}{\sigma_m} = \frac{\rho_p V_p^2 d_p}{\sigma_p}}_{2} \Rightarrow \frac{\sigma_m}{\sigma_p} = \left(\frac{V_m}{V_p}\right)^2 \times \frac{d_m}{d_p} \times \frac{\rho_m}{\rho_p} = \frac{1}{4} \times \frac{1}{4} \times 2 = \underbrace{\frac{1}{8}}_{1}$$