ID:-----------------	**Quiz: No. 5**	**Time: 20 minutes**
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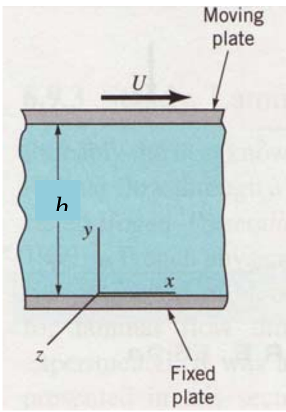
Two horizontal, infinite, parallel plates are spaced a distance *h* apart. A viscous liquid with viscosity μ is contained between the plates. The bottom plate is fixed and the upper plate moves parallel to the bottom plate with a velocity *U* in x direction. The pressure distribution *p(x)* is assumed to be *ax+b* in which *a*, and *b* are constant parameters. (a) Write up all assumptions you need to make. (b) List all boundary conditions. (c) Derive an expression for the x-component of velocity *u* as a function of *y*. (d) For which value of *a*, *u* at *h/2* is half of the upper plate velocity ($u\big|_{y=h/2} = U/2$)?



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Incompressible continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

x-component of the incompressible Navier–Stokes equation:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

y-component of the incompressible Navier–Stokes equation:

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

z-component of the incompressible Navier–Stokes equation:

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

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Solutions:

1. The flow is steady, i.e. any time derivative is zero. (0.5)
2. This is a parallel flow (the y component of velocity, v, is zero) and consequently the flow is fully developed based on continuity. (0.5)

x momentum:

$$\rho\left(\underbrace{\underbrace{\frac{\partial u}{\partial t}}_{=0;assumption1}}_{0.5}+\underbrace{u\underbrace{\frac{\partial u}{\partial x}}_{=0;continuity}}_{0.5}+\underbrace{v\underbrace{\frac{\partial u}{\partial y}}_{=0;assumption2}}_{0.5}\right)=\underbrace{\underbrace{-\frac{\partial p}{\partial x}}_{=-a}}_{0.5}+\underbrace{\underbrace{\rho g_x}_{=0}}_{0.5}+\mu\left(\underbrace{\underbrace{\frac{\partial^2 u}{\partial x^2}}_{=0;continuity}}_{0.5}+\underbrace{\underbrace{\frac{\partial^2 u}{\partial y^2}}_{unknown}}_{0.5}\right)$$

$$\Rightarrow -a+\mu\frac{\partial^2 u}{\partial y^2}=0$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2}=\frac{a}{\mu}\Rightarrow \frac{\partial u}{\partial y}=\frac{a}{\mu}+c_1$$

$$\Rightarrow \underbrace{u=\frac{a}{2\mu}y^2+c_1 y+c_2}_{1}$$

Boundary conditions: (2)

1. u at y=0 is 0 $\Rightarrow 0=\frac{a}{2\mu}\times 0+c_1\times 0+c_2\Rightarrow c_2=0$
2. u at y=h is U $\Rightarrow U=\frac{a}{2\mu}\times h^2+c_1\times h+0\Rightarrow c_1=\frac{U}{h}-\frac{a}{2\mu}\times h$

$$\Rightarrow \underbrace{u=\frac{a}{2\mu}y^2+\left(\frac{U}{h}-\frac{a}{2\mu}\times h\right)y}_{1}$$

Finally:

$$u\big|_{y=h/2}=\frac{a}{2\mu}\frac{h^2}{4}+\left(\frac{U}{h}-\frac{a}{2\mu}\times h\right)\frac{h}{2}=\underbrace{-\frac{a}{2\mu}\frac{h^2}{4}+\frac{U}{2}}_{0.5}$$

To have $u\big|_{y=h/2}=U/2$, a has to be zero $\Rightarrow a=0$ (1)