

Measurement of Kinematic Viscosity

Purpose

To measure the kinematic viscosity of a fluid, the uncertainty of the measurement, to compare the measured kinematic viscosity to the manufacturer's value, and to demonstrate the effects of viscosity by comparison of the fall times for spheres of different densities.

Design of the Experiment

A fluid deforms continuously under the action of a shear stress. The rate of strain in a fluid is proportional to the shear stress. The proportionality constant is the *dynamic viscosity* (μ). Viscosity is a thermodynamic property and varies with pressure and temperature. For a given state of pressure and temperature, there is a large range of values of viscosity between common fluids. For instance, there is a variation of three orders of magnitude between water and glycerin, the fluid which will be used in this experiment. Since the term μ/ρ , where ρ is the density of the fluid, frequently appears in the equations of fluid mechanics, it is given a special name, *kinematic viscosity* (ν).

The most common methods used to determine viscosity are the rotating-concentric-cylinder method (Engler viscosimeter) and the capillary-flow method (Saybolt viscosimeter). Alternatively, we will measure the kinematic viscosity through its effect on a falling object. The maximum velocity attained by an object in free fall (terminal velocity) is strongly affected by the viscosity of the fluid through which it is falling. When terminal velocity is attained, the body experiences no acceleration, so the forces acting on the body are in equilibrium (Figure 1).

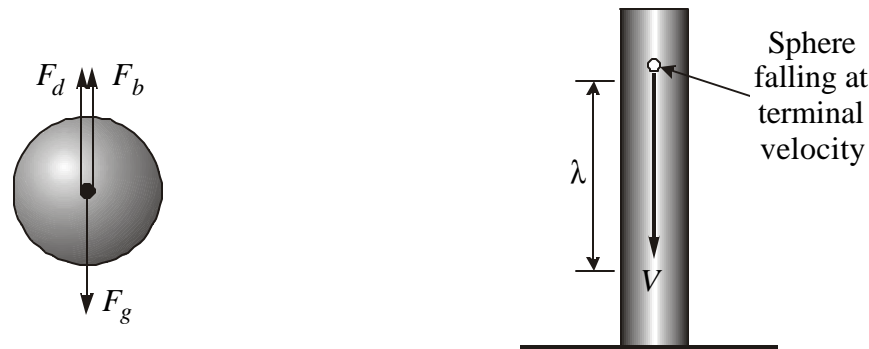


Figure 1. Experimental arrangement

The forces acting on the body are the gravitational force,

$$F_g = mg = \rho_{sphere} \frac{D^3}{6} g \quad (1)$$

a force due to buoyancy,

$$F_b = \rho_{fluid} \frac{D^3}{6} g \quad (2)$$

and the resistance of the fluid to the motion of the body, similar to friction. This force is described by the Stokes expression for the drag force on a sphere (valid for Reynolds numbers, $Re = VD/\nu \ll 1$),

$$F_d = 3 \rho_{fluid} \nu V D \quad (3)$$

where D is the sphere diameter, ρ_{fluid} is the density of the fluid, ρ_{sphere} is the density of the falling sphere, ν is the viscosity

of the fluid, F_d , F_b , and F_g , denote the drag, buoyancy, and weight forces, respectively, V is the velocity of the sphere through the fluid (in this case, the terminal velocity), and g is the acceleration due to gravity (White 1994).

Once terminal velocity is achieved, a summation of the vertical forces must balance. Equating the forces gives:

$$\mathbf{n} = \frac{D^2 g (\mathbf{r}_{sphere} / \mathbf{r}_{fluid} - 1) t}{18 \mathbf{I}} \quad (4)$$

where t is the time for the sphere to fall a vertical distance \mathbf{I} .

Using equation (4) for two different balls, namely, teflon and steel spheres, the following relationship for the density of the fluid is obtained, where subscripts s and t refer to the steel and teflon balls, respectively.

$$\mathbf{r}_{fluid} = \frac{D_t^2 t_t \mathbf{r}_t - D_s^2 t_s \mathbf{r}_s}{D_t^2 t_t - D_s^2 t_s} \quad (5)$$

Measurement Systems

In this experiment, we will allow a sphere to fall through a long transparent cylinder filled with the glycerin Figure 2. After the sphere has fallen a long enough distance so that it achieves terminal velocity, we will measure the length of time required for the sphere to fall through the distance, \mathbf{I} . Figure 3 illustrates the block diagram of the measurement systems and data reduction equations for the results

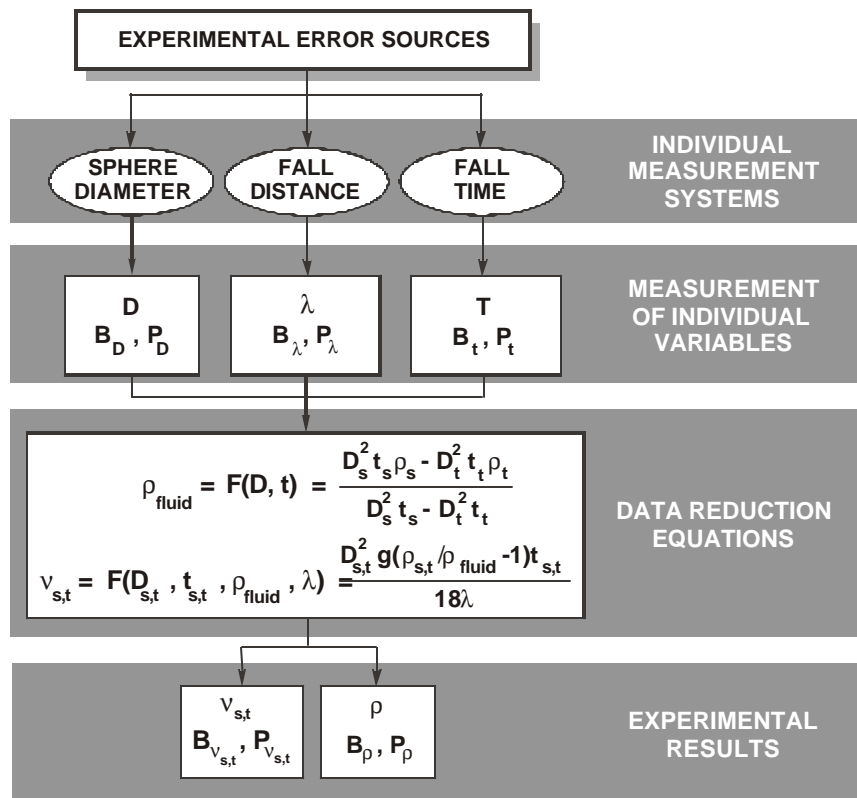


Figure 3. Block diagram of density/viscosity experiment including: measurement systems, data reduction equations, and results

The equipment (measurement systems) used here includes:

- A transparent cylinder (beaker) containing glycerin. A scale is attached to its side to read the distance the sphere has fallen.
- Teflon and steel spherical balls of different sizes

- Stopwatch
- Micrometer
- Thermometer

Measurement Procedures

1. Density and viscosity are functions of temperature. Measure the temperature of the room. We will use this temperature to compare our measurement with the manufacturer's data.
2. Two horizontal lines are marked on the vertical cylinder. We will measure the time required for the spheres to fall between these two lines. Measure the distance between the two lines, \mathbf{L} .
3. Measure the diameter of each sphere (teflon and steel) using the micrometer.
4. Release the sphere at the surface of the fluid in the cylinder. Then, release the gate handle.
5. Release the spheres, one by one, and measure the time for the sphere to travel the length \mathbf{L} .
6. Repeat steps 3- 5 for all spheres. At least 10 measurements will be taken for each type of spheres.

Since the fall time of the sphere is very short, it is important to measure the time as accurately as possible.

Start the stopwatch as soon as the bottom of the ball hits the first mark on the cylinder and stop it as soon as the bottom of the ball hits the second mark. Two people should cooperate in this measurement with one looking at the first mark and handling the stopwatch, and the other looking at the second mark. Both individuals should agree on the value of the measurement.

Uncertainty Analysis

The methodology for estimating uncertainties is according to the AIAA S-071-1995 Standard (AIAA, 1995) as summarized in Stern et al. (1999) for multiple tests ($M = 10$). The block diagram for propagation of errors in the measured density and viscosity is provided in Figure 3. The data reduction equations for density and viscosity of glycerin are equation (5) and (4), respectively. First, the elemental errors for each of the independent variable, X_i , in data reduction equations should be identified using the best available information (for bias errors) and repeated measurements (for precision errors). Table 1 contains the summary of the elemental errors assumed for the present experiment.

Table 1. Assessment of the bias limits for the independent variables

Bias limit	Bias Limit	Estimation
$B_D = B_{D_s} = B_{D_t}$	0.000005 m	½instrument resolution
$B_t = B_{t_s} = B_{t_t}$	0.01 s	Last significant digit
B_L	0.00079 m	½instrument resolution

The bias limit, precision limit, and overall uncertainty for the experimental results, namely the density and viscosity of glycerin, are then found using Eqs. (14), (23) and (24) in Stern et al. (1999). Note that in the present analysis we will neglect the contribution of the correlated bias errors in equation (14).

Density of glycerin

The total uncertainty for the density measurement is:

$$U_{\bar{r}_G} = \sqrt{B_{r_G}^2 + P_{\bar{r}_G}^2} \quad (6)$$

The bias limit B_{r_G} , and the precision limit $P_{\bar{r}_G}$, for the result are given by:

$$B_{r_G}^2 = \sum_{i=1}^j \mathbf{q}_i^2 B_i^2 = \mathbf{q}_{D_t}^2 B_{D_t}^2 + \mathbf{q}_{t_t}^2 B_{t_t}^2 + \mathbf{q}_{D_s}^2 B_{D_s}^2 + \mathbf{q}_{t_s}^2 B_{t_s}^2 \quad (7)$$

$$P_{\bar{r}_G} = \frac{2 \cdot S_{\bar{r}_G}}{\sqrt{M}} \quad (8)$$

where the sensitivity coefficients (calculated using mean values from previous data for the independent variables) are:

$$\mathbf{q}_{D_t} = \frac{\partial \mathbf{r}_G}{\partial D_t} = \frac{2 D_s^2 t_t t_s D_t (\mathbf{r}_S - \mathbf{r}_t)}{[D_t^2 t_t - D_s^2 t_s]^2} = 296,808.22 \frac{kg}{m^4} \quad \mathbf{q}_t = \frac{\partial \mathbf{r}_G}{\partial t_t} = \frac{D_s^2 D_t^2 t_s (\mathbf{r}_S - \mathbf{r}_t)}{[D_t^2 t_t - D_s^2 t_s]^2} = 30.6 \frac{kg}{m^3 \cdot s}$$

$$\mathbf{q}_{D_s} = \frac{\partial \mathbf{r}_G}{\partial D_s} = \frac{2 D_t^2 t_t t_s D_s (\mathbf{r}_t - \mathbf{r}_S)}{[D_t^2 t_t - D_s^2 t_s]^2} = -527,208.8 \frac{kg}{m^4} \quad \mathbf{q}_{t_s} = \frac{\partial \mathbf{r}_G}{\partial t_s} = \frac{D_s^2 D_t^2 t_t (\mathbf{r}_t - \mathbf{r}_S)}{[D_t^2 t_t - D_s^2 t_s]^2} = -78.1 \frac{kg}{m^3 \cdot s}$$

The standard deviation for density of glycerin for the 10 repeated measurements is calculated using the following formula

$$S_{\bar{r}_G} = \left[\sum_{k=1}^M \frac{(\mathbf{r}_k - \bar{\mathbf{r}})^2}{M-1} \right]^{1/2}$$

Viscosity of glycerin

Uncertainty assessment for the glycerin viscosity will be based on the measurements conducted with the teflon spheres. Selection of the teflon spheres experiment is based on a better agreement with Stokes' law requirements ($Re \ll 1$). The total uncertainty for the viscosity measurement is given by equation (24) in Stern et al. (1999):

$$U_{\bar{n}} = \sqrt{B_{\bar{n}}^2 + P_{\bar{n}}^2} \quad (9)$$

The bias limit $B_{\bar{n}}$, and the precision limit $P_{\bar{n}}$, for viscosity (neglecting correlated bias errors) is given by equations (14) and (23) in Stern et al. (1999), respectively.

$$B_{\bar{n}}^2 = \sum_{i=1}^j \mathbf{q}_i^2 B_i^2 = \mathbf{q}_{D_t}^2 B_{D_t}^2 + \mathbf{q}_t^2 B_t^2 + \mathbf{q}_{r_G}^2 B_{r_G}^2 + \mathbf{q}_I^2 B_I^2 \quad (10)$$

$$P_{\bar{n}} = \frac{2 \cdot S_{\bar{n}}}{\sqrt{M}} \quad (11)$$

The bias limits for B_{D_t} , B_t , B_r were evaluated previously in conjunction with the estimation of U_{r_G} . B_I is provided in Table 1. The sensitivity coefficients, \mathbf{q} , (calculated using mean values from previous data for the independent variables) are:

$$\mathbf{q}_{D_t} = \frac{\partial \mathbf{n}}{\partial D_t} = \frac{2 D_t^2 g \left(\frac{\mathbf{r}_t}{\mathbf{r}_G} - 1 \right) t_t}{18 \mathbf{I}} = 0.2202 \frac{m}{s} \quad \mathbf{q}_t = \frac{\partial \mathbf{n}}{\partial t} = \frac{D_t^2 g \left(\frac{\mathbf{r}_t}{\mathbf{r}_G} - 1 \right)}{18 \mathbf{I}} = 2.27 \times 10^{-5} \frac{m^2}{s^2}$$

$$\mathbf{q}_{r_G} = \frac{\partial \mathbf{n}}{\partial r_G} = \frac{D_t^2 g r_t t_t}{18 \mathbf{I} r_G^2} = 1.36 \times 10^{-6} \frac{m^5}{kg \cdot s} \quad \mathbf{q}_I = \frac{\partial \mathbf{n}}{\partial I} = -\frac{D_t^2 g \left(\frac{\mathbf{r}_t}{\mathbf{r}_G} - 1 \right) t_t}{18 \mathbf{I}^2} = -1.15 \times 10^{-3} \frac{m}{s}$$

The standard deviation for the viscosity of glycerin for the 10 repeated measurements is calculated using the following formula

$$S_{\bar{n}} = \left[\sum_{k=1}^M \frac{(n_k - \bar{n})^2}{M - 1} \right]^{1/2}$$

Data Analysis

During the data acquisition, values for the following quantities should be obtained.

$$T = \underline{\hspace{2cm}} ; \quad \rho_s = \underline{7991 \text{ kg/m}^3} ; \quad I = \underline{\hspace{2cm}}$$

$$g = \underline{\hspace{2cm}} ; \quad \rho_f = \underline{2148 \text{ kg/m}^3}$$

Suggested table for measurements

Trial T= 26.4 °C I = 0.61 m	TEFLON		STEEL		RESULTS	
	D_t	t_t	D_s	t_s	r	n
	(m)	(sec)	(m)	(sec)	(kg/m ³)	(m ² /s)
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
Average						
Std.Dev. (S_i)						

Data reduction includes the following steps

1. Calculate the fluid density for each measurement using equation (5).
2. Calculate the kinematic viscosity for each measurement using equation (4) for either sphere type.

Calculate the uncertainties for the experimentally determined glycerin density and kinematic viscosity. Compare the measured values to the manufacturer's values. Figure 3 contains benchmark data provided by the glycerin manufacturer.

Discussions

1. How does the size of the sphere affect the viscosity?
2. How does the drag coefficient $C_D = F_D / (1/2) \rho V^2 A$ vary with viscosity, where $V \gg I/t$ is the velocity of the sphere?
3. How does viscosity change with temperature?
4. We have taken the contributions to uncertainty from the correlated bias error terms to be zero. For this experiment, is this a good assumption?
5. We have ignored the contributions to uncertainty from sources other than time and distance. If the sphere had not achieved terminal velocity, how would this affect the measurement? Is this a bias or precision error? Would this error make our value for viscosity larger or smaller?

6. The Stokes relation is valid only for $Re \ll 1$. The Reynolds number (a dimensionless number that characterizes the flow) is given by $(l V / \nu)$ where l and V are characteristic length and velocity scales for the flow. Calculate the Reynolds number for our experiment. Are we within the limits?

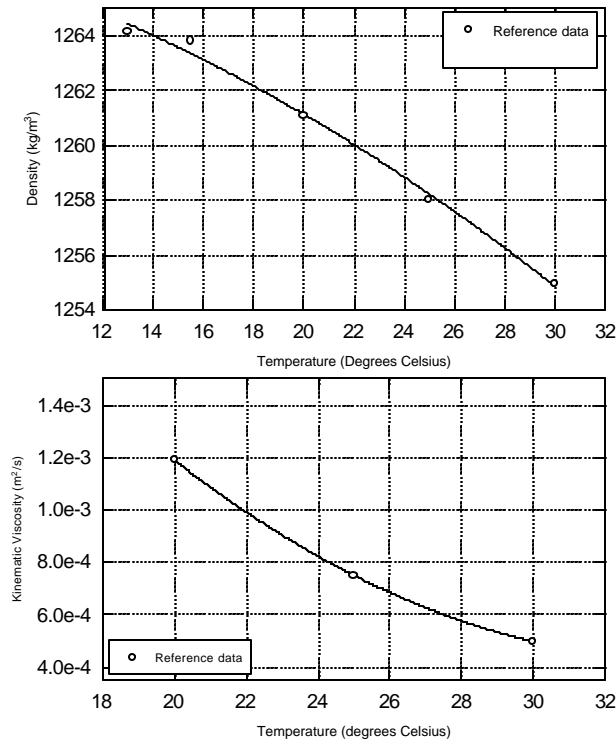


Figure 3. Reference data for the density and viscosity of 100% aqueous glycerin solutions (Proctor & Gamble Co., Product Catalogue, 1995)

References

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