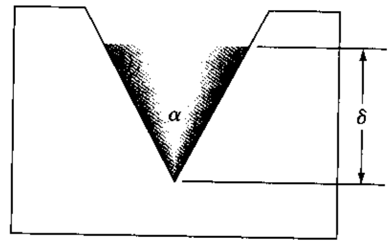
P5.69 A simple flow measurement device for streams and channels is a notch, of angle α, cut into the side of a dam, as shown in Fig. P5.69. The volume flow Q depends only on α, the acceleration of gravity g, and the height δ of the upstream water surface above the notch vertex. Tests of a model notch, of angle $\alpha = 55°$, yield the following flow rate data:

δ, cm	10	20	30	40
Q, m^3/h	8	47	126	263

(*a*) Find a dimensionless correlation for the data (*b*) Use the model data to predict the flow rate of a prototype notch, also of angle $\alpha = 55°$, when the upstream height δ is 3.2 m.



P5.69

$F(Q, g, \delta, \alpha) = 0$ $\quad n = 4 \quad m = 2 \quad r = n - m = 2$

$\quad L^3/s \quad L/s^2 \quad L$

$f(\Pi_1, \Pi_2) = 0$

By inspection $\Pi_1 = \alpha$ (already nondimensional)

$$\Pi_2 = Q^a g^b \delta^c$$
$$= Q g^{-1/2} \delta^{-5/2} = \frac{Q}{g^{1/2}\delta^{5/2}}$$

$\Pi_2 = f(\Pi_1)$

For Π_1 = constant = 55° use model data to get f = constant

$\Pi_2 = .23$ for $\delta = 10, 20, 30,$ and 40 cm

$$Q_p = \sqrt{g}\, \delta_p^{5/2} \times .23$$
$$= \sqrt{9.81} \times (3.2)^{5/2} \times .23$$
$$= 13.2 \text{ m}^3/\text{s}$$

Model Ship Testing:

$$C_T(Re, Fr) = C_W(Fr) + C_V(Re) \qquad \text{Froude Scaling}$$

$$C_{W_p} = C_{W_m} = C_{T_m} - C_{V_m}(1+k)$$

$$C_{T_p} = C_{W_p} + C_{V_p}(1+k)$$

C_{T_m} measured in towing tank & $C_V(Re)$ based on model-ship correlation line, ie, data for flat plate skin friction for $Re_m \le Re \le Re_p$. k = form factor, which accounts for 3D effects on C_V and usually assumed independent Fr and Re. Note C = drag coefficient $= \dfrac{F}{\frac{1}{2}\rho V^2 S}$

Example: $L_p = 100$ m, $V_p = 10$ m/s, $S_p = 300\ m^2$

$\alpha = 1/25$, $F_{T_m}(V_m) = 60$ N

Find V_m using Fr scaling:

$$Fr_m = \frac{V_m}{\sqrt{gL_m}} = Fr_p = \frac{V_p}{\sqrt{gL_p}}$$

$$V_m = V_p\sqrt{\frac{L_m}{L_p}} = V_p\sqrt{\alpha}$$

$$= 10\sqrt{1/25} = 2 \text{ m/s}$$

$$Re_m = \frac{U_m L_m}{\nu} = \frac{2 \times 100/25}{10^{-6}} = 8 \times 10^6$$

$\nu = 10^{-6}\ m^2/s$

$$Re_p = \frac{U_p L_p}{\nu} = \frac{10 \times 100}{10^{-6}} = 10^9$$

$\rho = 1000\ kg/m^3$

Use ITTC model-ship correlation or flat plate friction lines, as per text

$C_{vm} = C_v(Re_m) = .003$ take $k=0$

$C_{vp} = C_v(Re_p) = .0015$

$F_{v_m} = \frac{1}{2}\rho U_m^2 S_m C_{vm}$ $S_m/S_p = \alpha^2$

$= \frac{1}{2} \times 1000 \times 2^2 \times (300/25^2) \times .003$

$= 2.88\ N$

$F_{w_m} = F_{Tm} - F_{vm} = 60 - 2.88 = 57.12\ N$

$$F_{wp} = \frac{1}{2}\rho U_p^2 S_p \times \frac{F_{w_m}}{\frac{1}{2}\rho U_m^2 S_m} = \alpha^{-3} F_{w_m}$$

$= 25^3 F_{w_m}$

$= 8.92 \times 10^5\ N$

$F_{vp} = \frac{1}{2}\rho U_p^2 S_p C_{vp} = \frac{1}{2} \times 1000 \times 10^2 \times 300 \times .0015$

$= .225 \times 10^5\ N$

$F_{Tp} = 9.14 \times 10^5\ N$

2.5% over estimate!

If neglect $C_v(Re)$: $F_{Tp} = \frac{1}{2}\rho U_p^2 S_p \times \frac{F_{Tm}}{\frac{1}{2}\rho U_m^2 S_m} = \alpha^{-3} F_{Tm} = 9.37 \times 10^5$